problem set 4, due December 19.

1. Two firms produce a single good using labor. The first firm requires l_1 units of labor to produce a unit of the good; the second firm needs l_2. Each firm can hire any amount of labor at a wage normalized to 1. If firms offer different prices, households buy only from the low-price firm. Each firm correctly believes that at the price p households will buy an amount p^{-2} of the good.
(a) Let $l_1 = l_2 = .75$. If the two firms form a cartel that sets prices and output levels for both firms so as to maximize the sum of their profits, what price does the cartel choose?
(b) Find the Bertrand equilibrium prices for $l_1 = l_2 = .75$ and for $l_1 = .7$ and $l_2 = .8$
(c) Find the Cournot equilibrium output levels when $l_1 = .7$ and $l_2 = .8$. (Hint: Solve first for the aggregate output and the corresponding equilibrium price. Then use this price to calculate each firm's market share.)

2. Capitalists hire workers to spend a day at the farm. If a worker works while she's there, she produces one unit of corn. If she shirks, she produces nothing, and she faces a probability p that the capitalist will detect her shirking and fire her. A fired worker collects an unemployment benefit b. A worker's expected utility from working at the wage w is $w - a$, her expected utility from shirking is $(1 - p)w + pb$, and she chooses the action that gives her the greater expected utility.
(a) Each capitalist sets a wage to maximize her profits. Express the profit-maximizing wage as a function of p, a, and b.
(b) Now capitalists can hire up to 1 unit of guard labor per production worker. Guard labor also carries a disutility of a for the worker who performs it. If a capitalist employs guards in a proportion m to her production workers, and if some production or guard worker shirks, the shirker is detected and fired with probability m. Solve for the m that maximizes capitalists' profits as a function of a and b.
(c) Suppose that capitalists run the profit-maximizing effort-extraction policies of (b). Let $b > a$. What happens to the aggregate productivity of employed labor as the unemployment benefit is increased?