

### problem set 3, due November 7.

Consider a circulating-capital model of the production of corn and steel. The production of a unit of corn uses up  $a_{11}$  units of corn and  $a_{21}$  units of steel, and it requires  $l_1$  units of labor. Unit output of steel uses up  $a_{12}$  units of corn and  $a_{22}$  units of steel, and it requires  $l_2$  units of labor.

(a) Wages, paid at the start of the period, are equalized at  $w$ , and profit rates on outlays for produced inputs and labor are equalized at  $r$ . Write down equations relating  $w$ ,  $r$ , and the prices of corn and steel. Explain carefully how these can be solved for relative prices and a profit rate as functions of a given wage. (Don't bother actually solving them.)

(b) Let

$$\begin{pmatrix} a_{11} \\ a_{21} \\ l_1 \end{pmatrix} = \begin{pmatrix} .5 \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} a_{12} \\ a_{22} \\ l_2 \end{pmatrix} = \begin{pmatrix} .2 \\ .4 \\ 2 \end{pmatrix}.$$

Write down profit-rate equalizing price equations for this economy. Find the wage-profit relation and graph it. Solve for the price of steel in terms of corn as a function of the corn wage. What is the maximum wage if profit rates can't be negative? What is the maximum profit rate if wages can't be negative?

(c) A second corn-producing activity is discovered that uses .6 units of corn and .5 units of labor. Profit-maximizing capitalists are free to choose between this activity and the corn-producing activity of (b). Solve for the wage-profit relation and plot it on your graph from (b). Find a switchpoint of this technology, a value of the wage such that capitalists switch activities as the wage passes through that value. Which activities are operated when the wage equals .1? Which activities are operated when the wage equals .4? What are the profit rate and the corn price of steel in each case?

(d) The price systems of (a)-(c) are made determinate by taking either the profit rate or the real wage as an exogenous variable and solving for relative prices and the remaining distributive variable. What, if any, plausible economic relationships are represented by imposing a given real wage as a boundary condition? What, if any, plausible economic relationships are represented by imposing a given profit rate as a boundary condition?