problem set 2, due October 17.

Three people are trading two goods for consumption. Each person has the Cobb-Douglas preferences that you studied in question 3 of problem set 1, represented by

$$u(x_1, x_2) = a \ln x_1 + (1 - a) \ln x_2. \tag{1}$$

Person A starts with 3 units of x_1 and 1 unit of x_2; person B has 1 unit of x_1 and 2 units of x_2; person C has 4 units of x_1 and 1 unit of x_2.

(a) Using your answer to question 3 of problem set 1, write each person's demand for good 1 as a function of the price p of good 1 in terms of good 2. Then find the economy-wide excess demand function for good 1. Graph it.

(b) Let $a = .5$ and solve for the Walrasian equilibrium relative price and allocation.

(c) A "Walrasian auction" is held that goes like this: The auctioneer announces a price for good 1 in terms of good 2. The two consumers report their demand for good 1 at this price. If good 1 is in excess demand, the auctioneer raises its price. If it's in excess supply, the auctioneer lowers the price. No actual trade takes place unless the excess demand is zero. Use your graph from (a) to show that this price-adjustment process converges to the equilibrium price that you computed in (b). Is this auction a good representation of the decentralized exchange of commodities?

(d) Calculate A's initial and final wealth by evaluating her initial and final holdings in the equilibrium prices. Do the same for B and C. Hint: You should find that A's wealth equals B's wealth before and after trade, and that A and B are poorer than C before and after trade. Briefly explain and evaluate the claim that these calculations show that wealth inequality is exogenous to the process of market exchange.

(e) The equilibrium allocation from (b) is in place. A fourth person, D, offers the following deal. Combining everyone's stocks of the two goods, D will add 3 units of good 1 and subtract 2 units of good 2. Then A, B, and C are free to redistribute the resulting stocks however they like. Will this deal allow for a Pareto improvement? Hint: Use the equilibrium prices to evaluate the deal.