

## problem set 1, due October 3.

1. One year of a worker's labor using one unit of corn planted at the start of the year produces two units of corn available at the end of the year.
  - (a) Express this technology by writing down the corn and labor inputs required per unit of corn produced.
  - (b) Capitalists sign year-long contracts for workers' labor, with wages paid as a stock of corn at the start of the year. Find the relation between the corn wage of labor and the capitalist's rate of profit on seed and wage corn. Express it first by writing the profit rate as a function of the wage, then by writing the wage as a function of the profit rate, and then by graphing the relation in wage-profit rate space.
  - (c) A government enforces a minimum corn wage of .5, and competition equalizes the wages paid by all capitalists at this minimum. Find the corresponding profit rate. The government raises the minimum wage to .6. Find the new profit rate.
  - (d) The calculation in (c) is an example of *comparative statics*. Which variables are treated as exogenous in that calculation? Which as endogenous?
  - (e) Choose one of the exogenous variables and explain verbally how you might change the model so that this magnitude is determined endogenously in it.

2. Jack's preferences over beer,  $x_b$ , and cigarettes,  $x_c$ , are represented by a function

$$u(x_b, x_c) = \min(x_b, x_c) \tag{1}$$

and Jill's preferences by

$$v(x_b, x_c) = x_b + x_c. \tag{2}$$

- (a) Draw a family of indifference curves for Jack and a family of indifference curves for Jill.
- (b) Jack has 4 beers and 10 cigarettes. Jill has 8 beers and 2 cigarettes. Locate this allocation in an Edgeworth box, and draw the corresponding better sets for Jack and for Jill. Show the set of trades that would make both Jack and Jill better off.
- (c) Find the set of exchange equilibria for the situation described in (b) and draw it in your Edgeworth box. Which equilibrium does Jack like best? Which does Jill like best?

3. A person's preferences over goods  $x_1$  and  $x_2$  are represented by

$$u(x_1, x_2) = a \ln x_1 + (1 - a) \ln x_2, \tag{3}$$

and she owns  $h$  units of the first good.

- (a) Write down the budget constraint that governs her purchases of the two goods when the price of good 1 in terms of good 2 is  $p$ .
- (b) Write down the problem of choosing the affordable consumption bundle that she prefers to all the others that she can afford at  $p$ .
- (c) Find the consumption bundle that solves this problem as a function of  $p$ . Check that your answer requires the marginal rate of substitution between the goods to equal their relative price.
- (d) Let  $h = 5$  and  $a = .4$ . How does the utility-maximizing consumption bundle change as  $p$  increases from 1 to 2? Draw a diagram in  $x_1, x_2$  space to depict this change.