THE WAGE–WAGE– . . . -WAGE–PROFIT RELATION IN A MULTISECTOR BARGAINING ECONOMY

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ABSTRACT

The equalization of profit rates across industries subject to firm-level bargaining over wages generates an interindustry wage structure with higher wages in capital-intensive sectors. The familiar inverse wage–profit relation gives way to a wage–wage– . . . -wage–profit surface on which the profit rate can vary directly with the wage paid in an individual industry. Institutional changes that decrease workers’ bargaining power and increase the incomes of the unemployed tend to compress the wage distribution; these changes draw political support from cross-class coalitions of low-wage workers and capital-intensive firms. Some capital-using, labor-saving technical changes that raise capitalists’ profits in current prices lower the equilibrium profit rate.

1. INTRODUCTION

Economists who spend time with wage regressions report that statistically visible differences between workers can explain no more than 30 per cent of the variation in cross-sections of the workers’ wages (Mortensen, 2003). Industry-level wage differentials capture a great part of the remaining dispersion, and orderings of industries by the wages paid in them are surprisingly resilient over time and across national economies (Krueger and Summers, 1988; Gittleman and Wolff, 1993).

This paper is about the wage structures generated by bargaining in a Leontief circulating-capital economy. I suppose that wages are set in Nash bargains between transiently matched workers and capitalists, and I consider systems of goods prices and wages that equalize rates of profit over all sectors of production.

This idealization is interesting from at least four points of view. For one thing it is an easy place in which to make the point that if many goods are

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produced by goods and labor, a simple bargaining mechanism is enough to send indistinguishable workers home with different wages. In one-sector models equilibrium wage dispersion tends to be cooked up from some mixture of imperfect competition in the market for the one good, a dispersion of firms’ technologies inside the frontier of efficient production, firm-level differences in the parameters of labor monitoring or training, and the strategic differentiation of wage offers by employers competing for scarce workers to fill vacant jobs. The capitalists of this paper are by contrast price takers with access to a common technology who face identically structured bargaining situations in which such head-hunting rivalries play no part.

The paper is also a development of the analysis of stationary price systems for linear production models, or prices of production. The original point of studying these systems was to better understand the institutionally variable joint determination of prices, wages and profitability. But that understanding calls for explicit models of institutionally variable wage-setting mechanisms. The model of this paper is one example.

Along with the class-wide interests that workers share with other workers or that capitalists share with other capitalists, workers and capitalists have interests special to the industries in which they work or invest that pit them against their class fellows in other industries. The third thing I do in the paper is to pinpoint some ways in which this kind of intra-class conflict interacts with the conflict between classes. I show that the equilibrium wages of workers in some industries can vary inversely with other workers’ wages and directly with the uniform profit rate, I distinguish cases in which profitability bears an increasing relation to wage inequality from cases in which that relation is decreasing, and I identify conditions under which institutional changes that variously compress or decompress the equilibrium wage distribution and raise or lower the equilibrium profit rate might be championed by cross-class coalitions made up of particular sections of the two classes.

Finally the wage mechanism of this paper might matter to an economy’s direction of technical change. I give one example in the line opened by Okishio (1961), showing that in this bargaining closure of the price-of-production system, in contrast to the uniform-wage case that Okishio considered, innovations that raise profits in current prices can result in a lower equilibrium rate of profit.

2. ‘THE DOUBLE CHARACTER OF THE WAGE’

Consider some capitalists who run activities from a Leontief technology described by a couple \((A, I)\). \(A\), there, is a semipositive, indecomposable,
productive \( n \times n \) matrix whose \( j \)th column \( a' \) lists the quantities of produced inputs needed to produce a unit of the \( j \)th good; \( l \), a positive row \( n \)-vector whose \( j \)th coordinate \( l_j \) gives the \( j \)th activity’s unit labor requirement. Each capitalist enters a production period owning stocks of commodities produced in the previous period, chooses a production plan that maximizes profits subject to a budget constraint in those stocks and prices \( p \), and buys the required commodity inputs.

She also tries to hire the required labor in a market for costlessly enforceable one-period employment contracts with wages to be paid at the end of the period. This market closes after a single round of matching, so matched workers and capitalists who fail to agree on a wage rate are out of work or business for the period. Let \( p_j \) and \( w_j \) be the price of the \( j \)th good and the wage paid in the \( j \)th activity. A capitalist who plans to run that activity takes \( p_j l_j^{-1} - w_j \) per worker if production goes ahead. If not, and because I assume that there are no asset markets outside the production sector in which she might invest this wealth, her fallback is the value of the inputs that she had planned to tie up with the worker, \( pa_j l_j^{-1} \). Capitalistically unemployed workers receive a payment \( v \), the outside wage, which might be understood as an unemployment benefit or as income available from economic activity outside the capitalist sector.

For some \( \beta \) in \([0, 1)\)—call this weight workers’ power, and let it take the same value in every activity—a generalized Nash bargain maximizes the weighted joint surplus over non-production

\[
(w_j - v)^\beta [(p_j - pa') l_j^{-1} - w_j]^{1-\beta}
\]

In the Appendix I show that the \( j \)th activity must then pay each worker

\[
w_j = \beta(p_j - pa') l_j^{-1} + (1-\beta)v
\]

a weighted average of the activity’s value-added per worker and the outside wage.

The bargaining assumptions that lead to (1) are a just-so story. It is possible to skip that story and to understand (1) in a more abstract spirit as standing in for any bargaining process that splits the difference between ceilings given by capitalists’ revenues net of material input costs and an economy-wide wage floor. For example, it might be taken as a rough-and-ready representation of enterprise- or industry-level collective bargaining subject to a uniform welfare or strike benefit. From this point of view the important content of (1) is that economy-wide institutional factors can influence wage setting in two distinct and in principle independently variable
ways: by raising or lowering the common floor and by making it easier or harder for workers to hold onto some of the local surplus.

Where $v$ is identified with a subsistence wage, (1) conjures Sraffa’s view of the ‘double character of the wage’ as including ‘besides the ever-present element of subsistence . . . a share of the surplus product’. Sraffa himself expresses this idea by working with an exogenous and uniform real wage measured in units of the economy’s given net output—in effect a given share of wages in national income—which assumption leaves him no way in which ‘to separate the two components of the wage’ (Sraffa, 1960, p. 9). The wage bargains (1) bring the Sraffian ‘double character’ of wages back to the surface.\(^1\)

Although of course it is crazy to suppose that workers take the same share in every sector, this has an analytical rationale as well. Cross-firm heterogeneity in the bargaining power of workers is ruled out as a source of the wage inequality that arises in this model. And institutional innovations that strengthen or weaken workers’ positions in bargaining economy-wide—new labor law, for example, or breakthroughs in national organizing by workers or employers—can be represented as perturbations of the parameter $\beta$.

I should call attention to one crucial contrivance, however. My assumptions that employment is transient and that workers and capitalists cannot return to the market to search for other production partners in the current period have the effect of insulating the wage bargain from the market’s degree of tightness or slack. An obvious next step is to remove that insulation. But in opening the price-of-production system to endogenous wage dispersion an interesting first step is to choose the smallest changes that possibly accommodate this. It is in such a spirit of analytical gradualism that (1) preserves that system’s signature decomposition between prices and quantities.

3. PRICES OF PRODUCTION

Faced with stationary prices $p$, capitalists who anticipate the wage bargains (1) are indifferent between committing capital to the different activities if and only if those prices satisfy

\[^1\] See Franke (1982) and Burgstaller (1994, pp. 78–87) for developments of Sraffa’s wage-share closure. I thank Duncan Foley for suggesting that I spell out the connection to Sraffa.
\[ p_j = (1 + r) p_a^j + w_j l_j \]  
\[ = (1 - \beta + r) p_a^j + \beta p_j + (1 - \beta) v l_j \]  
\[ = \left[ 1 + r (1 - \beta)^{-1} \right] p_a^j + v l_j \]  
for some \( r \geq 0 \). So a price system that supports the production of all \( n \) goods by profit-maximizing capitalists and that takes a working-class consumption basket \( d \) as numéraire can be written compactly as

\[ p = (1 + \rho) p A + v l \]  
\[ p d = 1 \]  
with the actual profit rate achieved by the capitalists related to the number \( \rho \) as in

\[ r = (1 - \beta) \rho \]  
This can be fleshed out by showing that some vector of activity levels clears all the goods markets at these prices on one or another assumption about capitalists’ and workers’ tastes. I will not do this since all the structure I need is in the wage, price and profit system, (1, 4, 5, 6), which follows from profit-rate equalization under the Nash bargaining rule whatever the quantity relations imposed on it.

Let

\[ v_{\text{max}} \equiv \frac{1}{l [1 - A]^{-1} d} \]  
It is shown in the Appendix that for any \( v \) in \([0, v_{\text{max}}]\) there is a unique positive \( p \) and \( \rho \) that solve (4, 5). I will write the solution for \( \rho \) as a function \( \rho(v) \) of the outside wage; I show in the Appendix that \( \rho'(v) < 0 \). Then the corresponding profit rate has

\[ \frac{\partial r}{\partial \beta} = -\rho(v) < 0 \]  
and

\[ \frac{\partial r}{\partial v} = (1 - \beta) \rho'(v) < 0 \]  
This establishes the following proposition.
Proposition 1: For any \((\beta, v)\) in \([0, 1) \times [0, v_{\text{max}}]\) there is a unique price-of-production bargaining equilibrium with a non-negative profit rate that is strictly decreasing in \(\beta\) and in \(v\).

This negative dependence of profitability on workers’ power and the outside wage recalls the inverse wage–profit relation that holds across the equal-profit-rate equilibria of those uniform-wage models. But in this model of unequal wages we have to consider, not a curve in the \(r, w\) plane, but a parametric surface, the wage–wage–...–wage–profit relation.

4. RELATIVE WAGES AND CAPITAL–LABOR RATIOS

One conclusion about the industry structure of wages is available right away. Substituting for \(p_j\) from (3) and for \((1 - \beta)^1r\) from (6) into (1) gives that

\[
w_j = \beta p(v) p a^1 l_j^1 + v
\]

For any two operated activities \(j\) and \(k\), then,

\[
w_j - w_k = \beta p(v) [p a^1 l_j^1 - p a^1 l_k^1]
\]

implying the following proposition.

Proposition 2: Wage differences in the bargaining equilibrium are proportional to differences in the activities’ ratios of the value-of-produced-inputs to labor employed.

This is a starting point for explaining the positive estimates of coefficients on capital intensities that are an outstanding result of interindustry wage regressions (Gittleman and Wolff, 1993; Araï, 2003; and for related explanations see Acemoglu, 1999 or Botwinick, 1993). And in a possible explanation of the stability of actual wage structures, Harrod-neutral technical change, by preserving the rank order of sectors’ capital–labor ratios, would preserve

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\(^2\) Interindustry wage inequality in these casual labor markets is consistent with equal expected personal incomes: if every worker faces the same stationary probabilities of being hired for the different activities, every sufficiently long-lived worker can expect to pass through high- and low-wage jobs in the same proportions. To give wage dispersion some political bite, assume instead that a worker has a greater probability of being hired for some activity in a later period if she is employed on it now.
their ordering by wages, too. I plan to pursue these explanations in another paper, but I think that their promise is reason enough to reconsider the comparative statics of income distribution in multisector economies on the assumption that wages are dispersed by bargaining as in (11).

5. VARIATION OF WORKERS’ POWER

Since prices are independent of $\beta$, it follows by differentiation of (10) that

$$\frac{\partial w_j}{\partial \beta} = \rho(v)p^a/l_j^1 > 0$$

(12)

Wages increase, and the profit rate falls, as workers’ power rises; along constant-$v$ sections of the equilibrium surface, wages and the profit rate are inversely related just as in the case of a uniform wage. In fact the first line of (2) makes it clear that

$$\frac{dw_j}{dr} = -\rho a/l_j^1 = \text{constant}$$

(13)

In this limited respect the linear wage–profit relation of Ricardo’s corn model is recovered without resort to a standard commodity or a labor theory of value. If the capitalists run the various production activities at levels $x$ that are independent of wage and profit rates, the national net income $p[I - A]x$ is also invariant with respect to $\beta$, so from (12) wages’ share in the national income is increasing in that parameter.

Consider next the wage structure. A pure increase in bargaining power amplifies any existing wage differences since by (11)

$$\frac{\partial}{\partial \beta}(w_j - w_k) = \rho(v)[p^a/l_j^1 - p^a/l_k^1]$$

(14)

the right-hand side is positive or negative according as $j$ has the greater or the lesser wage. So any measure of wage inequality that is increasing in the absolute values of these differences is increasing in $\beta$. Greater bargaining power for workers that raises wages across the board comes at the cost of greater inequality among workers, and there follows proposition 3.

Proposition 3: Institutional changes that increase $\beta$ while leaving $v$ unchanged increase all wages and all interindustry wage differences. Wage levels and the profit rate vary inversely on a constant-$v$ section of the equilibrium surface, as do the profit rate and wage inequality.
6. VARIATION OF THE OUTSIDE WAGE

I turn to comparisons of the equilibria picked out by different values of the outside wage at a constant value of workers’ power. From (10)

\[ \frac{\partial w_j}{\partial v} = I_j^{-1} \beta \left[ \rho'(v) \rho a^j + \rho(v) \frac{\partial p}{\partial v} a^j \right] + 1 \]  

(15)

It follows that

\[ \lim_{(\beta,v) \to (1,v_{\max})} \frac{\partial w_j}{\partial v} = \rho'(v) \rho a^j / I_j^{-1} + 1 \]  

(16)

And in the Appendix I show that

\[ \min_j \rho a^j / I_j^{-1} < -\rho'(v) < \max_j \rho a^j / I_j^{-1} \]  

(17)

wherever the outermost expressions are not in fact equal. Barring the fluke of equal equilibrium capital intensities, then, the right-hand side of (16) is strictly negative for at least one activity. So for \((\beta, v)\) in some \([\bar{\beta}, 1] \times [\bar{v}, v_{\max}]\) the equilibrium wage in the sector with the locally greatest value of capital per head is decreasing in the outside wage. And since the profit rate is everywhere decreasing in \(v\), it follows that in this parametric region it is true of at least one sector that the sector-specific wage–profit relation induced by a pure variation in the outside wage slopes up.

This ambiguous behavior of individual wages nonetheless washes out of the comparative statics of the aggregate wage share. From (1) wages’ share in the national income when capitalists run activities at the intensities \(x\) is

\[ \omega = \sum_j w_j / p[I - A] x = \frac{\beta p[I - A] x + (1 - \beta) v l x}{p[I - A] x} = \beta + (1 - \beta) \frac{v l x}{p[I - A] x} \]  

(18)

If \(x\) is constant, it follows that

\[ \frac{\partial \omega}{\partial v} = (1 - \beta) \frac{l x}{p[I - A] x} \left\{ 1 - \frac{v}{p[I - A] x} \frac{\partial p}{\partial v} [I - A] x \right\} \]  

(19)

The Appendix shows that the term in brackets is certainly positive and hence that the wage share is increasing in the outside wage.

Now consider the inequality of wages. Equation (15) implies that, for any \(j\) and \(k\),

\[ \frac{\partial}{\partial v} (w_j - w_k) = \beta \left\{ \rho'(v) (\rho a^j / I_j^{-1} - \rho a^k / I_k^{-1}) + \rho(v) \frac{\partial p}{\partial v} [a^j - a^k] \right\} \]  

(20)
Suppose the partial derivatives of the prices with respect to \( v \) were zero. Then the right-hand side of (20) is negative if and only if \( j \) has the greater value of capital per head and hence a greater wage. If these capital-revaluing price derivatives are absolutely small enough, then wage differences are decreasing in \( v \), and a pure variation in the outside wage induces an upsloping relation between capitalist profitability and the size of those differences. By the same token, if this capital-revaluation (‘price-Wicksell effect’) term is large and positive, there is nothing systematic to be said about wage dispersion’s dependence on the outside wage; analysis is frustrated by the same uncooperative behavior of relative prices that is the crux of the ‘Cambridge’ problem in capital theory.

Collecting these conclusions gives the following proposition.

**Proposition 4:** For great enough values of \( \beta \), an institutional change that increases \( v \) while leaving \( \beta \) unchanged, although it certainly increases the wage share of national income, can reduce the wage paid in at least one high-wage sector, so that the relation of that wage to the profit rate is positive along constant-\( \beta \) sections of the wage-wage-...-wage-profit surface. If capital revaluation effects are small, this change also decreases the size of interindustry wage differences, and wage inequality and the profit rate are positively related on those sections.

7. WORKING-CLASS CLEAVAGE

An old radical tradition holds that it is possible for privileged workers to join the capitalists in taking a surplus from the working class as a whole. In this section I use a counterfactual strategy for classifying working-class privilege, comparing how different groups of workers would fare in moving from that position to some interesting benchmarks.³

Take an economy described by some \((\beta, v)\), write \( r(\beta, v) \) for its equilibrium profit rate and \( w_j(\beta, v) \) for the equilibrium value of its \( j \)-th wage, and let

\[
\bar{w}(\beta, v) \equiv \frac{1}{1[1 - (1 + r(\beta, v))A]^{-1}d}
\]

so that \( r(0, \bar{w}(\beta, v)) = r(\beta, v) \): when wages are equalized at \( \bar{w}(\beta, v) \), the profit rate is unchanged from the original equilibrium for \( \beta, v \). If \( w_j(\beta, v) \) gives the value of the \( j \)-th wage in the equilibrium for \( \beta, v \), the Appendix shows that

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∃j,k : w_j(β, v) >  w(β, v) > w_k(β, v) \quad (22)

except where wages are already equal in the equilibrium for (β, v). So the project of equalizing wages without depressing the general profit rate necessarily cleaves the class into two opposed sections: one group who would gain from it and a second group of losers.

That is not too surprising. But now suppose that wages are to be equalized at the value

\[ w_{\text{max}} = \frac{1}{l[I-A]^{-1}d} \quad (23) \]

that sends the profit rate to zero. In the Appendix I show that

\[ \lim_{\beta \to 1} \max_j w_j(\beta, v) > \bar{w}_{\text{max}} > \lim_{\beta \to 1} \min_j w_j(\beta, v) \quad (24) \]

except where all wages are equal by a fluke. It follows that for any \( v < \bar{w}_{\text{max}} \) there is a \( \bar{\beta}(v) < 1 \) such that

\[ \beta \geq \bar{\beta}(v) \Rightarrow \exists j, k : w_j(\beta, v) > \bar{w}_{\text{max}} > w_k(\beta, v) \quad (25) \]

Workers in at least one industry are better off in a status quo marked by positive profits and dispersed wages than after an egalitarian socialist revolution that abolishes profits while leveling all wages. If they care only about their wages in equilibrium, these workers will side with the capitalists against the rest of their class.

Of course these kinds of equilibrium comparisons cannot bear much weight in explaining the course of struggle over institutional change. Even if these bargaining equilibria are asymptotically stable in a price and investment dynamics, people are unlikely to spend time computing the comparisons and likely to care most about how they fare in the transition to equilibrium. The next section steps around these difficulties by considering workers and capitalists who take a less Olympian view of their economic interests.

Before leaving (25), though, I should point out another way of reading it. The quantity \( l[I-A]^{-1}d \), the reciprocal of the maximum uniform wage measured in d-units, is also just the labor embodied in one unit of d bundles. And (25) says that for great enough \( \beta \), there are \( j \) and \( k \) with

\[ w_j(\beta, v)l[I-A]^{-1}d > 1 > w_k(\beta, v)l[I-A]^{-1}d \quad (26) \]

workers who have respectively more and less labor embodied in their wage bundles than they contribute to production. If \( \beta \) is big enough, bargaining
partitions the class into a Marxianly exploited stratum and a stratum of Marxian exploiters. Taking these points together gives the following proposition.

*Proposition 5*: Workers in at least one low-wage industry stand to gain and workers in at least one high-wage industry stand to lose from institutional change that equalizes wages while leaving the profit rate unchanged. If $\beta$ is big enough, workers in at least one high-wage industry stand to lose from a revolution that abolishes profits while equalizing wages; these workers are net Marxian exploiters.

8. INSTITUTIONAL INNOVATION

Consider an economy that is in the price-of-production bargaining equilibrium for some $(\beta, v)$ and suppose that workers and capitalists can act to bring about small changes in those parameters, for example by supporting legislation to raise or lower unemployment benefits (in the case of $v$) and to promote or curtail workers’ ability to strike (in the case of $\beta$). And suppose that these actors expect the prices of produced commodities to remain constant. This myopia excuses them from having to work out the equilibrium-displacing effects of their decisions. It also creates the possibility of interesting conflict among the capitalists, whose interests in equilibrium changes are by contrast identical.

Differentiating through (1) on the assumption that goods prices stay the same gives that

$$ dw_j = [(p_j - pa^j)^{-1} - v] d\beta + (1 - \beta) dv $$

The term in square brackets is just the surplus of value-added per worker over the outside wage, and so it equals $\rho(v)pa^j/l^j$. A myopic break-even line for wages or profits in the $j$th activity can thus be written as

$$ dv = -(1 - \beta)^{-1} \rho(v)pa^j/l^j d\beta $$

Capitalists in the $j$th sector should accept a small change in the direction given by $(d\beta, dv)$ if it lies below this line and should reject it otherwise, and the sector’s workers should follow the opposite policy.

Suppose that within each class political weights are distributed over the sectors and that a coalition of sectors all of whose members accept some institutional change under the myopic rule (28) can impose it on the remaining members of their class if they together account for more than half of their
class’s total weight. A given assignment of weights to the different sectors of capitalists thus picks out a class-wide break-even line such that all and only the changes below the line have the support of a dominant coalition. Likewise for the workers: all and only those innovations that lie above a class-wide break-even line can be imposed by some coalition. Since the slopes of sectoral break-even lines are given in (28) by the sectors’ capital–labor ratios, a class’s class-wide break-even line is steeper, the greater the weight of its more capital-intensive members. I will say that a class has a more or less capital-intensive center of gravity according as its class-wide break-even line has absolutely greater or lesser slope.

Now suppose that, to be viable, a change in $b$ and $v$ requires the support of dominant coalitions in both classes. The viable changes occupy a region below the capitalists’ break-even line and above the workers’ line. If the workers’ political center of gravity is more capital-intensive than the capitalists’ center so that their break-even line is steeper, the region of viable changes has $\beta$ increasing and $v$ decreasing. If the capitalists have the steeper line, the viable region lies in the opposite quadrant, with $\beta$ decreasing and $v$ increasing. The political alignment most favorable to a social-democratic Great Compression of wage rates, then, puts all the workers’ weight on the most labor-intensive sectors and all the capitalists’ weight on the most capital-intensive sectors, while the inverse polarization promotes wage-dispersing exchanges of greater workers’ power for a lower outside wage.

From the fact that $r = (1 - \beta)\rho(v)$ a break-even line for the equilibrium profit rate is given by

$$dv = -(1 - \beta)^{-1} \rho(v) [\rho'(v)]^{-1} d\beta$$

By (17) this lies between the myopic break-even lines (28) for the activities that are most and least capital-intensive in the equilibrium prices. So if capitalists have an intermediate center of gravity that coincides with (29), and if workers’ weight is concentrated on either the most capital-intensive activities or the most labor-intensive ones, all the viable changes raise the equilibrium profit rate. Profitability is best served by the combination of a middle-of-the-road capitalist coalition with either a capital- or a labor-intensive worker coalition. And it is served worst by the opposite scenario: where workers’ weight centers on the break-even line for equilibrium profitability and capitalists’ weight is skewed towards one or the other sectoral extreme, viable changes always bring down the general profit rate.

These claims come together in the following proposition.

**Proposition 6:** Viable institutional changes are wage-compressing (wage-dispersing) if the capitalists have a more (less) capital-intensive political
center of gravity than the workers. Viable institutional changes increase (decrease) the equilibrium profit rate if political weight in the capitalist (working) class is concentrated in intermediate sectors while political weight in the working (capitalist) class is skewed towards either capital- or labor-intensive sectors.

Although it is hard to say more at this level of abstraction, this proposition gives some idea of the explanatory pay-off to political-economy arguments that cross the two classes with $n$ sectors of production. The dependence of directions of change on sectoral political alignments that shows up here might help to account for the differential evolution of wage-setting systems.4 And since political arrangements that depress profitability are especially vulnerable to disruption, these alignments' induced effects on the general profit rate might help to explain their differential longevity.

9. COLLECTIVELY SELF-DEFEATING TECHNICAL CHANGE

Apart from regulating institutional evolution in these ways, the bargaining arrangements of this paper impinge on an economy's direction of technical change. This section presents one example of the difference bargaining can make.

In a circulating-capital model closed by an exogenously constant uniform real wage, the introduction of activities that raise capitalists' profit rates in current equilibrium prices necessarily induces a new equilibrium with a strictly greater rate of profit (Okishio, 1961). Wages and the profit rate are both endogenous to the equilibria I am discussing, so this constant-wage experiment is unavailable to me. What I can consider, though, are the displacements of equilibrium brought about by profit-maximizing technical change holding constant the distributive parameters ($\beta, v$). I will show that there is a class of technical changes that are profitable in the old prices but that lower the equilibrium profit rate when bargaining power and the outside wage are unchanged.

In this bargaining economy the technological upshot of the profit motive depends on the timing of wage bargains and technological learning. Making

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4 Ferguson (1984) and Swenson (2002) explain inter- and post-war capital–labor accords as the projects of specific sectoral coalitions, and Swenson argues that differences in the terms of these compromises in Sweden and the USA are explained in part by differences in the coalitions' industrial compositions.
an opportunistic choice from the wealth of plausible scenarios, I assume that innovation follows bargaining and is unanticipated by it.\textsuperscript{5} Suppose that a capitalist enters a period planning to run the activity \((a', l')\). She signs a costlessly enforceable contract for a worker’s labor at the wage given by (1) for the inherited technology. Before production can begin, however, she draws a prospective new activity and \((\vec{a}', \vec{l}')\) and must decide whether to substitute it for \((a', l')\) given the contractual value of the wage and the current equilibrium commodity prices \(p\).\textsuperscript{6} Her unit labor cost for the activity \((\vec{a}', \vec{l}')\) is

\[
\beta(p_j - pa')l_j^\gamma \vec{l}_j' + (1 - \beta)v\vec{l}_j'
\]

The new activity yields a higher return than the old one if and only if the price of the good it produces exceeds the sum of its material input cost marked up at the old profit rate and this unit labor cost, or

\[
p_j > (1 + r)p\vec{a}^\gamma + \beta(p_j - pa')l_j^\gamma \vec{l}_j' + (1 - \beta)v\vec{l}_j'
\]

Let the shadow cost of an activity \((a', l')\) be given by

\[
\alpha(a', l_j) \equiv (1 + \rho(v))pa' + vl_j
\]

This cost, which for the original \(j\)th activity just equals the equilibrium price of the \(j\)th good, values produced inputs at prices marked up by the factor \(1 + \rho(v)\) and labor at the outside wage \(v\).\textsuperscript{7} In the Appendix I show that any innovation that increases these shadow costs so that

\[
v(l_j - \vec{l}_j) < (1 + \rho(v))p\cdot(\vec{a}' - a')
\]

\textsuperscript{5} I mean that I am choosing these over the alternatives because they have the possibly interesting implication that profit-maximizing innovation can induce a lower equilibrium profit rate. You will want to keep reading the section if (1) you believe that capitalist growth has included episodes of declining profitability that invite a technological-cum-social explanation, or (2) you enjoy hearing stories about other people’s self-defeating behavior. I should point out that the myopia of the capitalists of the text is in the spirit of Okishio (1961), which studied the technological implications of current-period profit maximization in the expectation of unchanged wages. My discussion follows Roemer (1981)’s version of the Okishio argument.

\textsuperscript{6} So it must be that innovators can purchase the inputs they need for their new activities after innovating. This is awkward at best; I have not modeled the product market, so I have no way of showing that innovators will succeed in acquiring their new inputs at the old prices. This repurchasing is, however, consistent with section 2’s assumption that the market value of a capitalist’s initial input bundle forms her fallback option for the wage bargain. For that assumption does not require that the capitalist be stuck with the same physical inputs.

\textsuperscript{7} I should emphasize that this shadow cost is irrelevant to the individual capitalists’ innovation decisions. Its interest is only analytical: it predicts those decisions’ unintended collective impact on the general profit rate.
induces a lower equilibrium profit rate, and I prove the following proposition.

**Proposition 7:** For any given values of the distributive parameters and any given initial technology there is a set of new activities that satisfy both inequalities (30) and (32). The activities in this set have more expensive capital requirements but smaller labor requirements. The equilibrium profit rate for the technology formed by switching to an activity in this set is less than the initial equilibrium profit rate.

The capital-using, labor-saving technical changes that satisfy (30) and (32), although they earn a higher profit rate in the old prices, bring about a lower general profit rate in the new equilibrium.8 Bargaining is responsible for driving the wedge between actual and shadow costs, and in the Appendix I describe a condition under which the probability of self-defeating technical change is increasing in workers’ power $\beta$.

This argument leaves open the possibility that capitalists might revert to a discarded activity because it is more profitable in the new prices than the adopted activity. In the numerical economies that I have looked at, although some innovations that lower the equilibrium profit rate are unsustainable in that sense, others are indeed sustained in the new equilibrium even by profit-maximizing capitalists who remember their technological pasts.

Imagine that some capitalists draw one of these sustainable, profitability-depressing innovations. And suppose that the price-of-production equilibria are stable and that the convergence to them is fast. Then the capitalists would all be better off were they all to discard the innovation rather than maximize their own current returns by implementing it. But then it is also true that, whatever the other capitalists do, each does better to innovate. Technical change in this situation has the prisoners’ dilemma character that is often attributed to Marx’s own arguments but that has proven difficult to establish in his terms.9

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8 I have not given any reason to suppose that technical changes will indeed satisfy those inequalities over time, so this conclusion does not establish a tendency for the profit rate to fall.

9 Compare the argument of Foley (1986, pp. 136–9) and Franke (1999) that, where Okishio’s constant real wage gives way to a constant wage share, cost-reducing technical change can bring down the general profit rate. Instead of appealing to an aggregate boundary condition, the new argument follows the one-sector model of Skillman (1997) by making a specific bargaining mechanism responsible for the relevant changes in real wages.
10. CONCLUSION

I have argued that the equalization of profit rates across industries subject to firm-level bargaining over wages generates an interindustry wage structure with higher wages in the capital-intensive sectors bearing a possibly positive relation to the general profit rate. I have argued that institutional changes that decrease (or increase) workers’ bargaining power and increase (or decrease) the incomes of the unemployed will tend to compress (or decompress) the wage distribution, and I have claimed that these changes draw political support from cross-class coalitions of workers and capitalists. And I have argued that individual capitalists’ pursuit of higher profits in current prices through capital-using, labor-saving technical change can condemn the whole class to a lower general rate of return.

In making those arguments I have called attention to some ways in which technical and institutional innovation might reorganize the distribution of income between and within the two classes of the ‘classical’ (stationary-price, equal-profit-rate, linear) economy. Although it is hard to say much about that, we can say something. And a small change in the conception of wage setting makes it possible to say more.

APPENDIX

Derivation of (1)

It is necessary and sufficient for a maximum of the joint surplus that

\[ 0 = \beta (w_j - v)^{\beta - 1} \left[ (p_j - pa')l_j^{-1} - w_j \right]^{1-\beta} - (1 - \beta)(w_j - v)^{\beta} \left[ (p_j - pa')l_j^{-1} - w_j \right]^{-\beta} \]

which simplifies to

\[ 0 = \beta \left( \frac{(p_j - pa')l_j^{-1} - w_j}{w_j - v} \right) - 1 + \beta \]

(A1)

and so to (1).

Existence of a unique, positive solution to (4, 5) with \( \rho'(v) < 0 \)

Following Kurz and Salvadori (1995, pp. 100–1) I rewrite the system as

\[ p = (1 + \rho)pA + \nu p d l \]

(A3)
and so as
\[ pA[I - vdl]^{-1} = (1 + \rho)^{-1} p \]  
which has the form of an eigenvalue problem. It is readily checked that
\[ [I - v dl]^{-1} = I + \frac{v}{1 - vld} dl \]  
where this inverse exists, so for \( v \leq v_{\text{max}} \leq (ld)^{-1} \) the matrix \( A[I - v dl]^{-1} \) is semipositive and indecomposable. Let \( \lambda \) be its greatest eigenvalue. Then by a Perron–Frobenius theorem the unique positive solution to (4, 5) has \( \rho \) equal to \( \lambda^{-1} - 1 \) and \( p \) proportional to the corresponding left-hand eigenvector. Since a matrix’s maximum eigenvalue is increasing in its elements, \( \rho \) is by (A5) decreasing in \( v \). Setting \( \rho \) equal to 0 in (4) and solving for \( p \) gives
\[ p = vl[I - A]^{-1} \]  
and hence
\[ pd = l = vl[I - A]^{-1}d \]
So \( v \) must equal \( v_{\text{max}} \) when \( \rho = 0 \), confirming \( v_{\text{max}} \) as the upper bound for values of \( v \) that support positive profits.

Proof of (17)
From (4) the vector of partials of the prices with respect to the outside wage satisfies
\[ \frac{\partial p}{\partial v} = \rho'(v)pA + (1 + \rho(v))\frac{\partial p}{\partial v}A + l \]  
which can be written as
\[ \frac{\partial p}{\partial v} = \{\rho'(v)pA + l\}[I - (1 + \rho(v))A]^{-1} \]  
Dotting both sides of (A8) into the numéraire \( d \) gives
\[ \frac{\partial p}{\partial v}d = 0 = \{\rho'(v)pA + l\}[I - (1 + \rho(v))A]^{-1}d \]  
Using the numéraire condition and observing from (4) that prices satisfy
\[ p = v I (I - (1 + \rho(v)) A)^{-1} \]  
\hspace{1cm} \text{(A10)}

it is possible to rearrange (A9) as

\[ \rho'(v) = \frac{1}{v z d} \]  
\hspace{1cm} \text{(A11)}

where

\[ z = p A (I - (1 + \rho(v)) A)^{-1} \]  
\hspace{1cm} \text{(A12)}

Suppose the second inequality in (17) were false and the first true. Then by (A11)

\[ -(\rho'(v))^{-1} I = v z d I \geq p A \]  
\hspace{1cm} \text{(A13)}

Postmultiplying both sides of this inequality by the strictly positive \([I - (1 + \rho(v)) A]^{-1}\) gives

\[ z d p > z \]  
\hspace{1cm} \text{(A14)}

after substitutions into the left-hand side from (A10) and into the right from (A12). But dotting both sides into \(d\) produces the contradiction \(z d > z d\). A symmetrical argument shows that the first inequality cannot fail to hold if the second holds.

**Proof that the bracketed expression in (19) is positive**

Substitution from (A11) into (A8) shows that

\[ \frac{\partial p}{\partial v} = (-(v z d)^{-1} p A + I)(I - (1 + \rho(v)) A)^{-1} \]  
\hspace{1cm} \text{(A15)}

from which it follows via (A10) and (A12) that

\[ \frac{\partial p}{\partial v} = v^{-1} (- (z d)^{-1} z + p) \]  
\hspace{1cm} \text{(A16)}

Substituting for these price derivatives into the bracketed expression in (19) then gives

\[ 1 - \frac{v}{p(I - A)x} \frac{\partial p}{\partial v} [I - A] x = \frac{z[I - A] x}{(zd)p[I - A] x} > 0 \]  
\hspace{1cm} \text{(A17)}
Proof of (22)

Unless all wages are equal, at most one inequality in (22) can be false. Suppose then that \( w_j(\beta, v) \leq \bar{\omega}(\beta, v) \) for all \( j \) with \( w_k(\beta, v) < \bar{\omega}(\beta, v) \) for some \( k \). Then from (2)

\[
p - (1 + r(\beta, v))pA \leq \bar{w}(\beta, v)l
\]

Postmultiplying both sides by the strictly positive vector \([I - (1 + r(\beta, v))A]^{-1}d\)

\[
p[I - (1 + r(\beta, v))A][I - (1 + r(\beta, v))A]^{-1}d < \bar{w}(\beta, v)l[I - (1 + r(\beta, v))A]^{-1}d
\]

This implies a contradiction since the left-hand side equals 1 by the numéraire condition while the right-hand side equals 1 by the definition of \( \bar{\omega}(\beta, v) \). A symmetrical argument dismisses the remaining possibility.

Proof of (24)

In the actual equilibrium

\[
w_j(\beta, v)I_j = \beta(p_j - pA^l) + (1 - \beta)vI_j
\]

so the vector of unit labor costs is

\[
\gamma(\beta, v) \equiv (w_1I_1, w_2I_2, \ldots, w_nI_n) = \beta p[I - A] + (1 - \beta)vI
\]

from which

\[
\gamma(\beta, v)[I - A]^{-1}d = \beta + (1 - \beta)\frac{v}{\bar{w}_{\max}}
\]

and so

\[
\lim_{\beta \to 1} \gamma(\beta, v)[I - A]^{-1}d = 1
\]

Unless all wages are equal, at most one inequality in (24) can be false. But if one were false it would follow that

\[
\lim_{\beta \to 1} (w_1I_1, w_2I_2, \ldots, w_nI_n)[I - A]^{-1}d \neq \bar{w}_{\max}l[I - A]^{-1}d
\]

contradicting (A23) since \( \bar{w}_{\max}l[I - A]^{-1}d = 1 \) by the definition of \( \bar{\omega}_{\max} \).
Proof of Proposition 7

I first show that the introduction of an activity satisfying (32) induces a lower equilibrium profit rate. Write the social technology formed by replacing $a'_{ij}$ and $l_j$ with $\bar{a}'_{ij}$ and $\bar{l}_j$ in $(A, l)$ as $(\bar{A}, \bar{l})$. By (32) and (4)

$$ (1 + \rho(v))p\bar{A} + v\bar{l} \geq (1 + \rho(v))pA + vl = p \quad (A25) $$

Subtracting $v\bar{l}$ from the outermost expressions and using the fact that $pd = 1$ gives

$$ (1 + \rho(v))p\bar{A} \geq p - vpd\bar{l} \quad (A26) $$

Taking $\lambda$ to be the Frobenius root of $A[I - vd\bar{l}]^{-1}$ and hence equal to $(1 + \rho(v))^{-1}$ as in the proof of proposition 1, it follows that

$$ p\bar{A}[I - vd\bar{l}]^{-1} \geq \lambda p \quad (A27) $$

provided that $v\bar{l}d \leq 1$. (If instead $v\bar{l}d > 1$, the new technology does not support a non-negative profit rate, so it is immediate that innovation decreases equilibrium profitability.) Let $m^i$ be the $i$th column of $\bar{A}[I - vd\bar{l}]^{-1}$. Then

$$ \text{for all } j, \frac{p\bar{m}^i}{p_j} \geq \lambda; \text{ for some } i, \frac{p\bar{m}^i}{p_i} > \lambda \quad (A28) $$

But it is a theorem on square matrices (Roemer, 1981, p. 110) that for any positive indecomposable $A$ and positive $q$, either $A$’s maximum eigenvalue $\lambda$ satisfies

$$ \max \frac{qa'}{q_i} > \lambda > \min \frac{qa'}{q_i} \quad (A29) $$

or these three expressions are equal. So writing the maximum root of $\bar{A}[I - vd\bar{l}]^{-1}$ as $\bar{\lambda}$, (A28) implies that if $\frac{p\bar{m}^i}{p_i} > \frac{p\bar{m}^j}{p_j}$ for some $i$ and $j$, then

$$ \bar{\lambda} > \min \frac{p\bar{m}^i}{p_j} \geq \lambda \quad (A30) $$

and that if instead $\frac{p\bar{m}^i}{p_i} = \frac{p\bar{m}^j}{p_j}$ for all $i$ and $j$, then

$$ \bar{\lambda} = \frac{p\bar{m}^i}{p_i} > \lambda \quad (A31) $$
It follows in either case that \( \bar{X} > \lambda \) and therefore that the new profit rate is less than the old.

I will now confirm that there are activities satisfying (32) and (30) and that these are capital-using and labor-saving in the sense that

\[
l_j - \bar{T}_j > 0 > p_j \{ a^j - \bar{a}^j \} \tag{A32}
\]

Substitution for \( p_j \) from (3) into (30) gives

\[
(1 - \beta l_j^{-1}\bar{T}_j) \left\{ \left[ 1 + (1 - \beta)^{-1}r \right] p a^j + v l_j \right\} - (1 + r) p a^j + \beta l_j^{-1}\bar{T}_j p a^j - v (1 - \beta) \bar{T}_j > 0
\]

With

\[
\eta \equiv \frac{\bar{T}_j}{l_j} \quad \text{and} \quad \mu \equiv \frac{p a^j}{p a^j} \tag{A34}
\]

this becomes

\[
\frac{vl_j (1 - \eta)}{pa^j} > \mu (1 + r) - \beta \eta - (1 - \beta) (1 + (1 - \beta)^{-1} r) \tag{A35}
\]

or

\[
\mu < \mu_l (\eta) \equiv \frac{vl_j (1 - \eta)}{(1 + r)pa^j} + \frac{1 + (1 - \beta) (1 - \beta)^{-1} r}{1 + r} \tag{A36}
\]

On the other hand (32) can be rewritten as

\[
[1 + (1 - \beta)^{-1} r] (\mu - 1) > \frac{vl_j (1 - \eta)}{pa^j} \tag{A37}
\]

or

\[
\mu > \mu_0 (\eta) \equiv \frac{vl_j (1 - \eta)}{[1 + (1 - \beta)^{-1} r]pa^j} + 1 \tag{A38}
\]

Evidently \( \mu_l (\eta) \) is greater or less than \( \mu_0 (\eta) \) according as \( \eta \) is less or greater than 1. So for any \( \eta \) in \((0, 1)\) there is an interval \((\mu_0 (\eta), \mu_l (\eta))\) with \( \mu_0 (\eta) > 1 \) such that for \( \mu \) in that interval \((\mu, \eta)\) satisfies both inequalities.

The probability of self-defeating innovation increasing in \( \beta \): a sufficient condition

The bounds \( \mu_l (\eta) \) and \( \mu_0 (\eta) \) are decreasing in \( \beta \), and therefore so must be

\[
\int_0^1 (\mu_l (\eta) - \mu_0 (\eta)) d\eta
\]
the area of the region in which both (32) and (30) hold. Because equilibrium commodity prices are independent of $\beta$, there is associated with every $(A, l, v, d)$ a mapping $\phi$ from the set of regions $T$ of $\mu, \eta$ space to the set of sets of prospective new activities

$$\phi T = \{(\bar{a}^j, l^j) | l^j = \eta^j, \bar{p}^j = \mu \bar{p}^j \text{ for some } j \text{ and for some } \mu, \eta \text{ in } T\}$$

(A39)

such that a switch to an activity in $\phi T$, evaluated in the equilibrium prices for $(A, l)$, yields proportional rates of labor-productivity and capital-cost change that live in $T$. If the probability measure describing the distribution of prospective new activities assigns a greater probability to $\phi T$, the greater the area of $T$, then the probability of drawing an innovation that satisfies (30) and (32) is increasing in $\beta$.

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