

Production decentralized by decentralized exchange

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1 Getting serious

Price theory so far is a theory of equilibria. There is no reason to study equilibria except where there's reason to believe that they're the stable rest points of dynamical systems representing decentralized exchange processes.

Economists have turned up only one general argument to that effect. It's shown in [1, 5, 10, 13] that a process of decentralized, voluntary trade that conserves an economy's aggregate goods stocks will converge to states in which all gains to trade are exhausted—the set (generically a continuum) of Pareto-efficient allocations that Pareto-dominate the initial position. (At least one of these exchange equilibria can be supported by a budget plane passing through the initial allocation, but in almost all cases [2] at most a finite number can; a generic exchange trajectory will come to rest somewhere away from this negligible “Walrasian” subset of the equilibrium set [5].)

That goods are conserved in trade means that production is out of the question. And the traders are simply people who like goods; unlike capitalist firms or wealthholders, they're not concerned with the profits they can make by making and selling goods or financing their manufacture. In this paper I discuss some possibilities for and some difficulties encountered in extending this argument to a model of capitalist production.

2 Production for exchange

Let the $n \times m_j$ input and output matrices \mathbf{A}_j and \mathbf{B}_j describe m_j production and storage activities available to the j th of z firms. Each firm holds stocks of the various goods to be used as inputs and expects to be able to sell its eventual outputs at prices \mathbf{p} . Given an n -vector of stocks \mathbf{k}_j and an n -vector of price guesses \mathbf{p} let $\mathbf{x}_j(\mathbf{k}_j, \mathbf{p})$ solve

$$\max_{\mathbf{x} \in \mathbb{R}_+^{m_j}} \mathbf{p} \mathbf{B}_j \mathbf{x} \text{ s.t. } \mathbf{A}_j \mathbf{x} \leq \mathbf{k}_j \quad (1)$$

and let $\lambda_j(\mathbf{k}_j, \mathbf{p})$ solve

$$\min_{\lambda \in \mathbb{R}_+^n} \lambda \mathbf{k}_j \text{ s.t. } \lambda \mathbf{A}_j \geq \mathbf{p} \mathbf{B}_j. \quad (2)$$

Write the common value of these dual programs as $v_j(\mathbf{k}_j, \mathbf{p})$ and, collecting the z firms' stocks in \mathbf{k} , collect the z value functions in $\mathbf{v}(\mathbf{k}, \mathbf{p})$.

3 Exchange for production

Let's talk about a trading process, represented by a family of operators $T_{\mathbf{p}}$ with parameter \mathbf{p} , that for given price expectations \mathbf{p} sends holdings \mathbf{k} to holdings $T_{\mathbf{p}}\mathbf{k}$. Defining the set of redistributions of \mathbf{k} as

$$Z(\mathbf{k}) \equiv \left\{ \mathbf{k}' \mid \sum_j \mathbf{k}'_j = \sum_j \mathbf{k}_j \right\}, \quad (3)$$

I'll accept any continuous $T_{\mathbf{p}}$ such that

$$T_{\mathbf{p}}\mathbf{k} \in Z(\mathbf{k}): \quad (4)$$

goods are neither created nor destroyed in trade;

$$\mathbf{v}(T_{\mathbf{p}}\mathbf{k}, \mathbf{p}) \geq \mathbf{v}(\mathbf{k}, \mathbf{p}): \quad (5)$$

firms refuse trades that lower the shadow value⁷ of their stocks;

$$T_{\mathbf{p}}\mathbf{k} \neq \mathbf{k} \Rightarrow \exists \mathbf{j}, \mathbf{v}_j((T_{\mathbf{p}}\mathbf{k})_j, \mathbf{p}) > \mathbf{v}_j(\mathbf{k}_j, \mathbf{p}): \quad (6)$$

trade takes place only if some firm gains by it; and, writing the τ th iterate of $T_{\mathbf{p}}$ as $T_{\mathbf{p}}^\tau$,

$$(\forall \tau, T_{\mathbf{p}}^\tau \mathbf{k} = \mathbf{k}) \Rightarrow \nexists \mathbf{k}' \in Z(\mathbf{k}), \mathbf{v}(\mathbf{k}', \mathbf{p}) \geq \mathbf{v}(\mathbf{k}, \mathbf{p}): \quad (7)$$

trade is permanently foregone only when the gains to trade are exhausted.

As Fisher remarks [4], trading processes of this kind, because they were discovered by Uzawa [12], are sometimes known as ‘‘Edgeworth processes’’ [cf. 3]. A \mathbf{k} that satisfies the consequent of (7) I'll call ‘‘a trading equilibrium for \mathbf{p} ’’.

4 Convergence to exchange equilibrium

Let

$$V_{\mathbf{p}}(\mathbf{k}) \equiv \sum_j v_j(\mathbf{k}_j, \mathbf{p}) \quad (8)$$

and notice that since $\{T_{\mathbf{p}}^\tau \mathbf{k}\}_{\tau=0}^\infty$ remains in $Z(\mathbf{k})$ there's a $\{T_{\mathbf{p}}^{\tau_v} \mathbf{k}\}_{v=0}^\infty$ that converges to some \mathbf{k}^* . By continuity this has

$$V_{\mathbf{p}}(\mathbf{k}^*) = V_{\mathbf{p}}\left(\lim_{v \rightarrow \infty} T_{\mathbf{p}}^{\tau_v} \mathbf{k}\right) = \lim_{v \rightarrow \infty} V_{\mathbf{p}}(T_{\mathbf{p}}^{\tau_v} \mathbf{k}) \quad (9)$$

If any such limit point \mathbf{k}^* were not an equilibrium, it would have to be that for some \mathbf{k}'

$$\exists \Upsilon : \forall \tau \geq \Upsilon, V_{\mathbf{p}}(T_{\mathbf{p}}^\tau \mathbf{k}^*) \geq V_{\mathbf{p}}(\mathbf{k}') > V_{\mathbf{p}}(\mathbf{k}^*) \quad (10)$$

a contradiction of the second equality in (9). So all the limit points of a trading path are equilibria, and every trading path approaches the equilibrium set.

5 A small open economy

The argument of the previous two sections is a trivial development of the convergence results for exchange economies that you have in [1, 5, 10, 13]. But let me point out some complications that crept in.

Those exchange economies are populated by households who try to trade their way toward preferred goods holdings. The crux of the convergence argument is that the utilities representing those preferences are generally nondecreasing and, in particular, increasing outside equilibrium. Section 4 arrived at a parallel conclusion by substituting for the household's $u(\cdot)$'s the similarly behaved $v(\mathbf{k}, \mathbf{p})$'s of the firms.

This required that I endow the firms with expectations about the prices at which they'd be able to sell their outputs. Some natural questions: Where do these expectations come from? And to whom are the firms expecting to sell the stuff?

The cleanest story you might tell is that this is a small open economy producing goods for sale at the world prices \mathbf{p} using externally untraded inputs that are traded internally according to $T_{\mathbf{p}}$. (Or \mathbf{p} might be the official prices set by a central planner while $T_{\mathbf{p}}$ describes illegal trade among enterprises.

But I'm after a more general picture. My thought is that the firms are planning to sell what they've produced to other firms in a new round of input trade. If so, \mathbf{p} is properly an expectation of exchange ratios in the next round of trading, and it should be updated to reflect the firm's experience of trading in earlier periods. I take this up in later work that studies long sequences of trade alternating with production.

6 The inefficiency of production organized by exchange

Like the exchange equilibria of the pure-exchange economy, these are Pareto efficient if the firms are regarded as the actors of concern: No firm can be made richer in the value of its expected output except by making some firm poorer. They are not however production-efficient in the sense of Koopmans: It might well be possible to increase the aggregate net output of some good at no cost to the production of any other good. For example one firm's technology might be strictly dominated by the production set of another firm. So long as the first firm retains some resources that are valuable in the final shadow prices, social production is inefficient.

This situation has a dual expression in the prices. While firms' shadow prices $\lambda_j(\mathbf{k}_j, \mathbf{p})$ are necessarily proportional in equilibrium, they may differ by a scale factor. Where they do differ in magnitude, production can be expanded by diverting a small amount of each good to the firm with the largest $\lambda_j(\mathbf{k}_j, \mathbf{p})$.

7 Arbitrage

Now distinguish a second class of actors, the capitalists, who own the good stocks deployed in production by the firms. Firms are joint ventures among capitalists who retain title to the inputs they commit and who share the profits in proportion to the values of their input contributions. A capitalist who lets the j th firm use her stock \mathbf{k}_c is thus entitled to a share $\lambda_j(\mathbf{k}_j, \mathbf{p}) \mathbf{k}_c / \lambda_j(\mathbf{k}_j, \mathbf{p}) \mathbf{k}_j$ of the value of the firm's output $\mathbf{p} \mathbf{B}_j \mathbf{x}(\mathbf{k}_j, \mathbf{p})$. Since $\mathbf{p} \mathbf{B}_j \mathbf{x}(\mathbf{k}_j, \mathbf{p}) = \lambda_j(\mathbf{k}_j, \mathbf{p}) \mathbf{k}_j$, this means the capitalist expects to take $\lambda_j(\mathbf{k}_j, \mathbf{p}) \mathbf{k}_c$. A profitmaximizing capitalist will try to move her stocks across the firms to raise the sum of their values in the firms' respective shadow prices.

Suppose that capitalists encounter opportunities to transfer stocks between firms, making a transfer only where this raises $\lambda_j(\mathbf{k}_j, \mathbf{p}) \mathbf{k}_c$. Respecifying T so as to represent this reallocation along with firms' input trading, I keep (4) and (6) but replace the remaining two conditions by

$$V_{\mathbf{p}}(T_{\mathbf{p}} \mathbf{k}) \geq V_{\mathbf{p}}(\mathbf{k}), \quad (11)$$

and

$$(\forall \tau, T_{\mathbf{p}}^{\tau} \mathbf{k} = \mathbf{k}) \Rightarrow \nexists \mathbf{k}' \in Z(\mathbf{k}), V_{\mathbf{p}}(\mathbf{k}', \mathbf{p}) \geq V_{\mathbf{p}}(\mathbf{k}). \quad (12)$$

The argument of section 3, which used only the behavior of the sum of valuations V , evidently carries over: every trajectory approaches the equilibrium set, now defined to satisfy the consequent of (12)

8 Encapsulation

Now firms' shadow prices are equal in equilibrium, there is a general shadow return factor $\mathbf{p}\mathbf{b}/\lambda^*\mathbf{a}$ equalized at 1 across all operated activities, and equilibrium production is Koopmans-efficient: in particular it maximizes the value of aggregate output in \mathbf{p} . The characteristic role of mobility/arbitrage, as distinct from the maximization of profits by the stock-constrained firms, is to bring about this systemwide equalization of rates of return in a common price system.

With arbitrage $V_{\mathbf{p}}(\mathbf{k})$ comes into its own as a thermodynamic-like potential [cf. 7] the surface of whose maxima picks out the set of macroscopic outcomes to which the system will relax under various macroscopic constraints. And this yields an informative duality between the system's extensive and intensive variables, its aggregate goods stocks $\bar{\mathbf{k}}$ and its equilibrium prices. Putting

$$V_{\mathbf{p}}^*(\bar{\mathbf{k}}) = \max_{\mathbf{k} \in Z(\bar{\mathbf{k}})} V_{\mathbf{p}}(\mathbf{k}) \quad (13)$$

you have

$$\frac{\partial V_{\mathbf{p}}^*(\bar{\mathbf{k}})}{\partial \bar{\mathbf{k}}} = \lambda^*. \quad (14)$$

Consider again the small-open-economy interpretation that I mentioned in section 5. If world prices \mathbf{p} prevail for the traded goods, trade or foreign direct investment in the previously untraded inputs will take place between two such economies if and only if their systems of equilibrium shadow prices differ. So the intensive variables play their usual thermodynamic role [cf. 12] of determining whether or not the extensive variables will be exchanged between two initially isolated subsystems.

Though there's generically a continuum of equilibrium configurations of the activities and stocks of the firms and capitalists, this is macroscopically degenerate in the class of economies for which a unique aggregate production plan supported by unique shadow prices yields the maximum value of aggregate output.

9 Labor and consumption

Now suppose that production requires labor. Along with the material coefficients \mathbf{B}_j and \mathbf{A}_j , the j th capitalist faces unit labor requirements \mathbf{l}_j . Let $v_j(\mathbf{k}_j, h_j)$ be the value of the dual programs

$$\max_{\mathbf{x} \in \mathfrak{R}_+^{m(j)}} \mathbf{p}\mathbf{B}\mathbf{x} \text{ s.t. } \mathbf{A}_j\mathbf{x} \leq \mathbf{k}_j, \mathbf{l}_j\mathbf{x} \leq h_j \quad (15)$$

and

$$\min_{\mathbf{p} \in \mathfrak{R}_+^n, w \in \mathfrak{R}} \mathbf{p}\mathbf{k}_j + wh_j \text{ s.t. } \mathbf{p}\mathbf{A}_j + w\mathbf{l}_j \geq \mathbf{p}\mathbf{B}. \quad (16)$$

Claims to labor power available for capitalist production or domestic production or consumption are traded along with produced goods. Working-class households indexed by s start out with a unit of labor power and no produced goods and act to raise the value of functions $u_s(\mathbf{k}_s, h_s)$. Let κ represent the capitalists' and workers' stocks of produced goods and claims to labor power and put $\omega(\mathbf{k}) \equiv \mathbf{v}(\mathbf{k}_j, h_j), u_1(\mathbf{k}_1, h_1), \dots, u_z(\mathbf{k}_z, h_z)$

Trade now follows an H that satisfies versions of (4,5,6,7) with H substituted for T , κ for \mathbf{k} , and ω for \mathbf{v} . The fixed points of H are again trading equilibria that are efficient in the sense that no redistribution raises the value of some actor's program without depressing some other actor's value. By an argument like that of section 3 trade paths approach arbitrarily close to the equilibrium set, and the values of capitalists' and workers' programs converge to some configuration $\omega^*(\kappa, H)$.

10 Edgeworth's indeterminacy and Sraffa's

Edgeworth's "indeterminacy of contract" [3] consists here in the fact that for given initial holdings κ , the asymptotic outcome of trade $\omega^*(\kappa, H)$ can vary quite radically across the equilibrium set depending on which of many admissible H 's obtains—depending, that's to say, on the laws, conventions, transactions technology, shopfloor regimes, collective-bargaining arrangements, etc. that govern trade and bargaining among capitalists and between capitalists and workers. The values of the workers' and the capitalists' programs are antagonistic across the equilibrium set: for example the j th capitalist's equilibrium profit rate can increase and all other profit rates remain constant only if the sum of workers' consumption values decreases.

So Edgeworth's theme encompasses the Sraffianish observation [11] that, for a given state of scarcity as reported in κ , the asymptotic price system and the asymptotic class income distribution given by $\omega^*(\kappa; H), \lambda^*, w^*$ depends on the correlation-of-class-forces facts rolled up in H .

11 Thanks

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12 References

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