Overtakable capitalist growth paths*

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August 25, 2005

abstract

Comparing a process of labor- and capital-augmenting technical change directed by capitalists' maximization of profits with a counterfactual in which decentralized innovation decisions are governed by noncapitalist property relations, I claim that if the two economies start from the same technology and capital stock there's a date T such that after T per-capita consumption is always strictly greater on the counterfactual.

Directed technical change, golden rule, growth-distribution duality

JEL numbers E25, O31, O41.

* I thank Duncan Foley for his comments on a draft.

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1 Introduction

Where production is organized for profit, profitability directs the evolution of production technology. In this paper I argue that typical paths of profit-directed technical progress, though certainly progressive, are in one respect *less* progressive than some other socially accessible paths. Comparing the development of a stylized capitalist economy with a counterfactual in which decentralized innovation decisions are governed by noncapitalist property relations, I claim that if the two systems are seeded with the same initial technology and capital stocks, there's a time T such that at each date after T the capitalist economy delivers less consumption per person than is afforded at that date on the counterfactual.

This argument continues an old tradition of using the duality of wage-profit and consumption-growth relations to expose the dynamic inefficiencies associated with consumption by a class of pure capitalists.[1] This theme, which goes back at least to Sato (1965), von Weizsäcker (1971), and Goodwin (1972), was recently retrieved by Thompson (2003). By bringing it to bear on problems of ongoing technical change, I'll show that its interest extends beyond the familiar problem of maximizing steady-state consumption subject to a stationary technology. Productive change is actually ongoing in capitalist economies, and the technical change mechanism that I discuss is a promising source of explanations for its historical profile, so I'm led to ask what that mechanism implies for the evaluation of our capitalist future.

2 Innovation directed by profitability

Consider a population of capitalists who hire labor and tie up nondepreciating stocks of a single good to produce more of that good. Time is continuous, and at any time t each capitalist runs a production activity described by a couple $(\rho(t), x(t))$, defined so that if $k(t)$ and $l(t)$ are the stock of the good

[1] For discussions of the choice of technique in light of growth-distribution duality, see von Weizsäcker (1971), Roemer (1977), or Craven (1979).

committed and the flow of labor employed, the activity yields a flow of the good equal to

$$\min\left(\rho\left(t\right)k\left(t\right),x\left(t\right)l\left(t\right)\right).$$

Every capitalist can employ any amount of labor at a wage $w(t)$. Holding $k(t)$ she maximizes profits by hiring $[\rho(t)/x(t)]\,k(t)$ so that she takes

$$r\left(t\right)=\rho\left(t\right)\left[1-w\left(t\right)/x\left(t\right)\right] \tag{1}$$

as the profit rate on her stock.

To collapse the population distribution of capitalists' technologies, I assume that the capitalists all follow the same rules for innovation and that they all start from a common technology. These assumptions allow me to reason about population aggregates by considering what happens to a single capitalist.

In particular I suppose that each capitalist's operated activity obeys

$$\dot{\rho}\left(t\right)=\chi\left(t\right)\rho\left(t\right)\ \text{ and }\ \dot{x}\left(t\right)=\gamma\left(t\right)x\left(t\right) \tag{2}$$

where at every moment t the capitalist chooses a profile of technical change $(\chi(t),\gamma(t))$ from an *innovation set*

$$I\equiv\left\{(\chi,\gamma)\in\Re^2\,|\gamma\leq g\left(\chi\right)\right\}$$

defined by a time-invariant, twice-continuously-differentiable, decreasing, strictly concave g. Writing the *wage share* as $\omega(t)\equiv w(t)/x(t)$ you have from (1) that for any $\omega\in[0,1]$ the capitalist can maximize the instantaneous rate of change of her profit rate by picking (χ,γ) in I to maximize

$$\dot{r}=\rho\left[\left(1-\omega\right)\chi+\omega\gamma\right], \tag{3}$$

a problem whose unique solution, characterized by the first-order condition

$$\omega g'\left(\chi\right)+1-\omega=0, \tag{4}$$

can be represented by differentiable, respectively decreasing and increasing functions of the wage share $(\chi(\omega), \gamma(\omega))$.

Due in its outlines to Kennedy (1964) and von Weizsäcker (1966) and lately revived by Duménil and Lévy (1995, 2003), Funk (2002), Acemoglu (2002), Foley (2003), and Julius (2005), this idealization of the innovation process gives pure expression to the idea that profitability acts as a social filter on technical change, selecting its tendential bias between labor and capital. I want to consider the resulting direction of productive change from a social point of view that I'll introduce in the next section.

3 Innovation to raise average consumption

If the capitalists operate the activity $(\rho(t), x(t))$ while building up their stocks according to

$$\dot{k}(t) = g(t) k(t), \tag{5}$$

their investment per employed worker is $[x(t)/\rho(t)]\, g(t)$, which leaves

$$c(t) = x(t)[1 - g(t)/\rho(t)] \tag{6}$$

as the ratio of aggregate consumption to employed labor. By rearranging this relation between growth rates and per-worker consumption as

$$g(t) = \rho(t)[1 - c(t)/x(t)] \tag{7}$$

you find that it's identical to the relation between profit and wage rates (1) pointed out in the last section.

A simple thought experiment will bring out the importance of this duality. Suppose that the human population of this economy is given by

$$N(t) = N_0 e^{nt}. \tag{8}$$

And imagine that production is to be locked into the activity that emerges from a period of technical progress directed by a benevolent engineer. If the economy with its eventually stationary technology is to maintain a constant ratio of employed workers to its total population, the stocks available to production must also eventually grow at the proportional rate n. Let the engineer choose a direction of technical change to maximize, at each moment, the rate of change of the level of consumption per capita that can be sustained on a path with a constant employment ratio. Defining the *investment weight*

$$\mu_n(t) \equiv \frac{n}{\rho(t)},$$

the engineer selects a $(\chi(t), \gamma(t))$ in I to maximize

$$\dot{c} = x\left[\mu_n \chi + (1 - \mu_n)\gamma\right].$$

The innovation that solves this problem can be written as $(\chi(1 - \mu_n), \gamma(1 - \mu_n))$ using the same functions that characterize the capitalists' decisions of section 1. It follows that the profit- and consumption-minded rules induce the same profile of technical change if and only if $r(t) = n$ so that the going shares of profits and wages $(1 - \omega(t), \omega(t))$ weigh the two components of innovation proportionally to the investment and consumption shares $(\mu_n(t), 1 - \mu_n(t))$. If the capitalists are to mimic the friendly engineer, distribution must respect "the golden rule" of Allais, Desrousseaux, Phelps, Robinson, Swan, and von Weizsäcker.[2] Golden-rule distribution is required, not so that the capitalists are led to choose a particular output-capital ratio from a given spectrum of techniques, but so that their exploration of new techniques is regulated by the right information.

4 Long-run neutralization of technical change

To this point I have only reworked in differential form some old ideas about a discrete one-time choice of technique. But I had promised to evaluate trajectories of ongoing technical change. This requires that I complete the model of section 2 with explicit dynamics for the wage and for capital accumulation.

[2] The bibliography in Phelps (1966) is a complete set of references to this multiply discovered result. Golden-rule reasoning is applied to the directed-technical-change model in chapter 8 of the Phelps book and in the final paragraph of von Weizsäcker (1966).

Let wages follow a real Phillips motion in the ratio of employed workers to people,

$$\upsilon(t) \equiv \frac{\rho(t)\, k(t)}{x(t)\, N(t)},$$

so that the wage share evolves according to

$$\dot{\omega} = \omega\left[\psi(\upsilon) - \gamma(\omega)\right] \tag{9}$$

for some $\psi(\upsilon)$ that has $\psi'(\upsilon) > 0$. If wages are entirely consumed, if each capitalist saves her profits in a constant proportion s, and if the population $N(t)$ grows as in (8), the employment ratio obeys

$$\dot{\upsilon} = \upsilon\left[s(1-\omega)\rho + \chi(\omega) - \gamma(\omega) - n\right] \tag{10}$$

while the augmentation of capital in technical change is governed by

$$\dot{\rho} = \chi(\omega)\rho. \tag{11}$$

The laws of motion $(9, 10, 11)$, first put together by Shah and Desai (1981) and reconsidered in Foley (2003), make a complete dynamical system in ω, υ, ρ. It turns out that by wedding endogenously directed technical change with the reserve-army wage-and-accumulation dynamics of Goodwin (1967) and Marx, this model arrives at a powerful explanation of the apparent long-run trendlessness of output-capital ratios and wage shares in growing capitalist economies.

To see how that goes, notice that since the boundary of the innovation set has $g'(0) < 0$, a unique ω^* between 0 and 1 induces *Harrod-neutral* technical change,

$$\chi(\omega^*) = 0. \tag{12}$$

Let $\gamma^* \equiv \gamma(\omega^*)$ be the associated rate of labor augmentation; I assume that

$$0 \leq \psi^{-1}(\gamma^*) \leq 1. \tag{13}$$

Then the flow picked out by $(9, 10, 11)$ has a locally asympotically stable steady state[3] with: a constant employment ratio equal to $\psi^{-1}(\gamma^*)$; a constant wage share whose value, ω^*, is invariant with respect to saving and population growth rates; Harrod-neutral progress at the rate γ^*; capital accumulation at the rate $g^* \equiv \gamma^* + n$; and capital productivity constant at

$$\rho_R^* \equiv \frac{g^*}{s(1-\omega^*)}. \tag{14}$$

Technical change, though it's intrinsically two-dimensional, degenerates to Harrod neutrality as the wage share approaches the value ω^* that annihilates its capital-augmenting component.

This model's fertility as a source of explanations for the Kaldorian regularities of capitalist growth paths invites attention to its implications for human wellbeing on those paths. Here is one. Should you find yourself in the capitalist steady state trying to maximize the instantaneous rate of change of consumption per head compatible with accumulation at the rate g^*, the reasoning of section 3 would instruct you to apply the weights

$$\frac{g^*}{\rho_R^*}, 1 - \frac{g^*}{\rho_R^*}$$

to capital and labor augmentation respectively. But positive capitalist consumption, in bounding the profit rate above g^*, bounds the profit share above the appropriate investment weight as from (14)

$$\frac{g^*}{\rho_R^*} = s(1-\omega^*) < 1 - \omega^*. \tag{15}$$

You can tell already that, viewed from the standpoint of social consumption, the attracting income distribution puts too much weight on the augmentation of produced material inputs and too little on the augmentation of labor power.

[3]For a proof of its local stability see Shah and Desai (1981).

5 Dueling filters

To bring this conclusion home I'll now compare the evolution of consumption per head in this system with an explicit consumption-driven alternative. No doubt this could be done by continuing section 3's fiction of the benevolent engineer, evaluating the development of $(9, 10, 11)$ against the benchmark of the centrally promulgated solution to some Bellman or Pontryagin problem.[4] However I think a more interesting comparison is to a decentralized economy in which innovation decisions are no better informed and no more powerfully computed than the capitalists' efforts.

For this reason I suppose that a group of co-ops, whose members exhaust a population that expands like (8), run production activities $(x_G(t), \rho_G(t))$.[5] At every moment the co-ops hold on loan from the state a stock of means of production on which they pay interest at the rate $i(t)$; the state also taxes co-ops' net income at some rate $\tau(t)$. The co-op workers consume what they produce net of these interest and tax payments, and each co-op chooses an innovation profile in I to maximize the rate of change in this consumption per worker due to innovation,

$$[1 - \tau(t)]\, x_G(t) \left[\mu(t)\, \chi + (1 - \mu(t))\, \gamma\right],$$

where

$$\mu(t) \equiv \frac{i(t)}{\rho_G(t)}$$

is the interest share of co-op revenue. The tax drops out of this problem, and the co-ops' constrained-best innovation is again described by $\chi(1-\mu), \gamma(1-\mu)$ with the consumption weight $1-\mu$ playing the same role in their decisions as the wage share plays in the capitalists' decisions. Finally I assume that the co-ops continually apply for, though they might not receive, loans sufficient to employ all their members on their evolving production activities and that the bank observes their aggregate demand for credit.

I'll suppose that the bank officials pursue full employment and golden accumulation with simple rules of

[4] Nordhaus (1967) studies the intertemporal optimization of directed technical change.

[5] The thought that coops might be carriers of the golden rule was suggested by remarks in Nell (1976); von Weizsäcker and Samuelson (1971) expound the regulating role of a golden interest rate in socialist planning; the classic golden-rule-minded comparison of socialist and capitalist growth paths is Nuti (1970).

thumb. In particular let them control the interest rate by a rule

$$\dot{i} = \alpha \left[-\chi \left(1 - \mu \right) + \gamma \left(1 - \mu \right) + n - i \right] i, \tag{16}$$

that corrects its deviation from the growth rate of co-ops' demand for loans with an intensity of adjustment $\alpha > 0$, giving rise to a differential equation for the investment weight,

$$\dot{\mu} = \left[- \left(1 + \alpha \right) \chi \left(1 - \mu \right) + \alpha \gamma \left(1 - \mu \right) + \alpha n - \alpha i \right] \mu. \tag{17}$$

And let the bank allow the stock on loan to the co-ops to accumulate at the proportional rate $i(t) + z(\upsilon(t))$ for some $z(\upsilon(t))$ with $z' < 0$ and $z(1) = 0$, setting the enterprise tax equal to $[i(t) + z(\upsilon(t))] / \rho(t)$ so that its new loans are covered by its interest and tax revenue. Being in fact credit-rationed so long as $\upsilon < 1$, the co-ops accept these loans. The bank's policy and the co-ops' innovation rules together yield

$$\dot{\upsilon} = \upsilon \left[z(\upsilon) + i + \chi \left(1 - \mu \right) - \gamma \left(1 - \mu \right) - n \right], \tag{18}$$

as a law of motion for the ratio of the employed to the total population.

Appendix A shows that the dynamical system $(16, 17, 18)$ has a unique globally asymptotically stable rest point characterized by full employment, Harrod-neutral progress at the rate γ^*, investment that accounts for a share $\mu^* = 1 - \omega^*$ of national spending, and interest and acccumulation rates constant at g^*. But the system approaches this configuration with a lower level of capital productivity,

$$\rho_G^* = \frac{g^*}{\mu^*} = \frac{g^*}{1 - \omega^*}, \tag{19}$$

than prevails in the capitalist long run, since it follows by comparison with (14) that if the capitalists do any consumption at all, $\rho_R^* > \rho_G^*$.

I don't mean to recommend the market syndicalism of $(16, 17, 18)$; *The Crime of Monsieur Lange* notwithstanding, I don't hold any particular affection for the co-op form. I've tried only to sketch a sanely informed decentralization of the social pursuit of higher consumption per head—a foil to $(9, 10, 11)$ that differs from it mainly in the decision rules that govern technical progress and in the property relations that give those rules currency.

In both economies ongoing technical change permits average social consumption to increase forever. And the unemployment-stabilizing rate of accumulation is endogenous rather than selected by the given path of labor supply. So I can't ask about these economies the question I asked in section 3; another criterion is called for. For any two growth paths, say that one path *overtakes* the second if there's a date T such that after T per capita consumption is always strictly greater on the first path. And for any two economies A and B characterized by possibly distinct decision rules for innovation and accumulation and by identical paths of labor supply and an identical innovation set, say that A overtakes B if every growth path of B is overtaken by a growth path of A that shares that B-path's technology and capital stock at some date 0. I'll now argue that the economy $(9, 10, 11)$ is indeed overtakable.

6 Overtakable capitalist growth paths

Though the nonlinearities in these systems preclude explicit comparisons of the time paths of consumption per head starting from arbitrary initial conditions, some indirect reasoning will establish overtaking in the most salient group of economies.

Suppose that at a time 0 a capitalist economy with $s < 1$ has already spent some time in its steady state. If labor productivity at 0 is x_0, consumption per employed worker after 0 is

$$c_R(t) = x_0 \left(1 - \frac{\gamma^* + n}{\rho_R^*}\right) e^{\gamma^* t} = x_0 \left(1 - s\left(1 - \omega^*\right)\right) e^{\gamma^* t}. \tag{20}$$

And consider an alternative history in which property relations are reset at 0: Co-ops usurp capitalists' role in the organization of production, and a state bank succeeds them in control of the social surplus, setting in motion the system $(16, 17, 18)$. This counterfactual path inherits a capital stock and technology that determine an initial value for the employment ratio, and the new bank authorities, having observed their Harrod-neutral capitalist prehistory, start the interest rate out at the established rate of accumulation g^*.

A glance at (16) and (17) shows that

$$i(t) \leq g^*, \mu(t) < \mu^* \Rightarrow \dot{i}(t) > 0 \tag{21}$$

and

$$i(t) > g^*, \mu(t) \geq \mu^* \Rightarrow \dot{\mu}(t) < 0. \tag{22}$$

An orbit that starts from

$$i(0) = g^*, \text{ and } \mu(0) = \frac{g^*}{\rho_R^*} = s\mu^* < \mu^*, \tag{23}$$

will thus have $i(t) > g^*$ and $\mu(t) < \mu^*$ throughout its approach to the rest point, and $\rho_G(t)$ converges to ρ_G^* monotonically from above with

$$\mu^* > \mu(t) \equiv \frac{i(t)}{\rho_G(t)} > \breve{\mu}(t) \equiv \frac{g^*}{\rho_G(t)} \tag{24}$$

along that trajectory.

The consumption per worker that's compatible with accumulation at the attracting rate g^*,

$$c_G(t) \equiv x(t)(1 - \breve{\mu}(t)),$$

itself increases at the proportional rate

$$\hat{c}_G \equiv \dot{c}_G / c_G = \frac{\breve{\mu}}{1 - \breve{\mu}} \chi(1 - \mu(t)) + \gamma(1 - \mu(t)). \tag{25}$$

Since from (20) $\hat{c}_R(t) \equiv \dot{c}_R / c_R = \gamma(1 - \mu^*)$, (25) implies that

$$\breve{\mu}(t)\chi(1 - \mu(t)) + (1 - \breve{\mu}(t))\gamma(1 - \mu(t)) > (1 - \breve{\mu}(t))\gamma(1 - \mu^*) \Rightarrow \hat{c}_G(t) > \hat{c}_R(t). \tag{26}$$

And the convexity of the innovation set implies that for any three investment weights μ_1, μ_2, μ_3,

$$\mu_1 > \mu_2 > \mu_3 \Rightarrow$$

$$\mu_1 \chi (1-\mu_2) + (1-\mu_1)\gamma(1-\mu_2) > \mu_1 \chi (1-\mu_3) + (1-\mu_1)\gamma(1-\mu_3). \tag{27}$$

But (24) says that μ^*, $\mu(t)$, and $\check{\mu}(t)$ satisfy the antecedent of (27), which in turn ensures that they satisfy the antecedent of (26) and hence that $\hat{c}_G(t) > \hat{c}_R(t)$ throughout the transition from a capitalist to a socialist steady state.

Putting

$$\eta(t) \equiv \frac{c_G(t)}{c_R(t)} \tag{28}$$

and noting that $\hat{c}_G(t) \to \hat{c}_R(t) = \gamma^*$ as capital augmentation dies out you get that

$$\lim_{t\to\infty} \eta(t) \equiv \eta^* > 1. \tag{29}$$

Rates of accumulation go to g^* in both systems, so consumption per person is asymptotically the sustainable consumption per worker, $c_R(t)$ or $c_G(t)$, scaled down by the employment ratio. But that employment ratio, equal to $\psi^{-1}[\gamma^*]$ in the capitalist steady state, converges to 1 in the socialist counterfactual. So the ratio of consumption levels per person on the two paths approaches $\eta^*/\psi^{-1}[\gamma^*] \geq \eta^* > 1$ as time goes to infinity.

I've now shown that the steady-state trajectories of capitalist economies whose capitalists consume some of their profits are overtaken by counterfactual paths formed by switching to the laws of motion $(16, 17, 18)$. Now suppose that the rest point of $(9, 10, 11)$ is globally asymptotically stable. Then the capitalist *economy* is overtakable, since any one of its orbits is overtaken by the identically initially conditioned orbit of a hypothetical economy that makes the switch to $(16, 17, 18)$ at a late-enough date.[6]

[6] Appendix B proves the last claim in the text. Since I have no proof of the global stability of $(9, 10, 11)$, the possibility remains that the system has nonconvergent paths which are not overtakable. I doubt that this qualification subtracts much economic interest from the current claim. The system $(9, 10, 11)$ only bears consideration insofar as its orbits spend most of their time near its steady state so that the model can recover the rough long-run constancy of wage shares and output-capital ratios. Periodic orbits can't be dismissed on these grounds, but simulations and bifurcation analysis have failed to detect any in this version of the directed technical change model.

7 Two objections

You might object that overtakability is an unsurprising consequence of the naïve linear saving rule that I've imposed on the capitalists. This objection loses some of its sting when it's recalled that people in the counterfactual economy are no more forward-looking in their saving behavior. But a more fundamental reply is that the same reasoning carries over to an economy of intertemporally-optimizing capitalists provided that its steady state has strictly positive capitalist consumption so that it continues to satisfy (15) where s is read as the endogenous steady-state ratio of saving to income.

Perhaps my thought experiment is unfair to the capitalist contender in a different way. The counterfactual economy has the state bank manipulating its tax and interest-rate policies to counteract deviations from full employment and the golden rule. This paper's capitalism, on the other hand, abstracts from government policy. To establish that property relations and not the absence of an active state are to blame for overtakability, should I not consider government policies in the capitalist case?

One possibility is a tax on the consumption expenditures of capitalist households. If capitalists are to hold positive wealth in a steady state, however, the prevention of overtaking requires that the tax be set so that the capitalists have zero steady-state consumption. A policy like that would be hard to sustain in a political economy where consumption-loving capitalist households enjoy anything like their actual political weight or the option of reinvesting their wealth in foreign production under more favorable tax rules. There's also a fair case to be made that this policy already constitutes a change in property relations since it extinguishes the ownership of capital goods as a source of effective claims on consumption.

A second policy is presumably easier to accommodate. Suppose the government announces that it will tax profit income at the rate τ. When the ratio of market wages to labor productivity is ω, the capitalists' after-tax share of income is $(1-\tau)(1-\omega)$. To rule out overtaking, the state should choose τ to satisfy

$$s(1-\tau)(1-\omega^*) = 1-\omega^* \tag{30}$$

so that the asymptotic investment share on the lefthand side equals the Harrod-neutrality-inducing before-tax profit share on the righthand side. But then the desired "tax",

$$\tau^* = 1 - s^{-1} < 0, \tag{31}$$

is in fact a profit *subsidy* that is more generous, the greater the capitalists' propensity to consume.

This result bears out the objection; overtaking is averted without starving the capitalists. But this conclusion only serves to clarify the social cost of capitalist consumption, which forces governments to choose between a permanent policy of redistribution from labor to capital and a permanent sacrifice of technologically feasible average consumption.

8 Late-capitalist growth and how to evaluate it

Beyond the direct conflict over the consumption of a given moment's product and the garden-variety tradeoff between current consumption and the possible future consumption afforded by savers' accumulation of capital, this argument calls attention to a third, specifically *evolutionary* axis of antagonism between capitalists' and social interests in consumption. If you take the empirically plausible view that capital productivity is declining in the earlier stages of capitalist development, and if you understand the recent growth of advanced capitalist countries as motion near the steady state of a profit-directed technical change system, you will conclude that, taxed as they are by capitalists' consumption, those economies have settled for Harrod neutrality and a retarded pace of labor productivity growth *too soon*.

This conclusion is a sort of bastard cousin of views often attributed to Marx. It's a fully spelled-out example of a scenario in which capitalist relations of production eventually fetter the development of the forces of production. Only a relative fettering is involved, however, not an absolute stagnation, and this need not generate any endogenous pressure for a change in production relations.[7] It's also striking that the scenario reverses the pattern of technical change that's supposed to bring about Marx's falling rate of

[7] Miller (1981) introduces and Cohen (1988) develops this relative/absolute fettering distinction in the context of Cohen's technologically determinist interpretation of Marx's historical materialism.

profit. Far from being destroyed by innovation of the capital-using, labor-saving type, late capitalism isn't getting *enough* of that kind of progress.

Though I won't be surprised if the empirical premises of this conclusion are rejected, I think the reasoning that leads up to it is independently interesting as an example of a currently underemployed strategy for evaluating long-run growth paths and the social arrangements that support them. The favorite current procedure asks whether a path succeeds or fails in maximizing an intertemporal social welfare function. The search for a defensible function to be maximized leads mostly to chagrin. For example it's not clear why future utilities should be discounted as is required for the convergence of utility integrals on infinite paths with perpetually positive consumption. (In fact it was this problem that inspired the authors of the original overtaking criterion to propose it; see for example von Weizsäcker (1965).) But another difficulty arises where some function like that is embraced: It doesn't tell us how to choose among the many decentralized economies whose actors have no hope of locating, let alone deliberately approaching, its maximum. This objection is especially serious where the long-run development of a technically progressive economy is concerned, since people can find out what kinds of technical changes are available only by going out and running their technologies, and since they have no way of reporting back their discoveries to a social planner.

The alternative I've been trying out goes like this. Suppose that certain qualitative hypotheses about the laws of motion under some institutional set-up are true. Can we conclude that people might rearrange their institutions in a way that makes them better off, using only information contained in their history of growth up through today and without contriving any new capacity for computing and imposing social plans? A conclusion like that is something to keep in mind.

Appendix A: global dynamics of (16, 17, 18)

I show that $(16, 17, 18)$ has a unique nonzero rest point that attracts all trajectories with strictly positive initial conditions. This system decomposes as $v(t)$ does not appear in $(16, 17)$. The $\chi(\cdot)$ terms in $(16, 17)$ vanish at μ^*, so both righthand sides are sent to zero by $\mu = \mu^*$ and $i = g^* \equiv \gamma(1-\mu^*) + n$. It's readily checked from (16, 17) that the isoclines of that subsystem have

$$\frac{di}{d\mu}\,|_{\dot{\mu}=0} < \frac{di}{d\mu}\,|_{\dot{i}=0} < 0 \tag{32}$$

everywhere, so that (μ^*, g^*) is the only interior rest point; that each isocline intersects both coordinate axes; and that both variables are increasing (decreasing) at all interior points below (above) their respective isoclines. Therefore the two open sets bounded by the isoclines and one of the axes are trapping regions in which all trajectories approach (μ^*, g^*), and every trajectory that originates in the positive orthant but outside these trapping regions must enter one of them or approach (μ^*, g^*) directly. Therefore $\mu(t), i(t) \to \mu^*, g^*$ from any interior initial position and so by (18) $\dot{\upsilon}(t)/\upsilon(t) \to z(\upsilon(t))$. But $z(\upsilon) > 0$ for $\upsilon < 1$. So $\upsilon(t) \to 1$, and the steady state is asymptotically stable in the large.

Appendix B: overtaking from arbitrary initial conditions

Suppose the rest point of (9, 10, 11) is globally asymptotically stable, and consider a counterfactual that originates at t_0 with initial conditions

$$i(t_0) = s(1 - \omega(t_0))\,\rho(t_0) \tag{33}$$

$$\mu(t_0) = s(1 - \omega(t_0)). \tag{34}$$

As $t_0 \to \infty$ you have that $\omega(t_0) \to \omega^*$, $\rho(t_0) \to \rho_R^*$, and therefore $i(t_0) \to g^*$. And since $s < 1$ there exists

$$\lim_{t_0 \to \infty} \mu(t_0) \equiv \bar{\mu} < \mu^*. \tag{35}$$

By choosing sufficiently small numbers ϵ and ε, you make it the case that any trajectory with initial conditions in

$$\left\{(i, \mu) \in \Re^2 \,||g^* - i| \leq \epsilon,\ |\bar{\mu} - \mu| \leq \varepsilon\right\}$$

has $i(t) \geq g^*$ for all t after some t_1 and $\mu(t) \leq \mu^*$ for all $t \geq t_0$. (35) and the convergence of $i(t_0)$ to g^* imply that, for big-enough t_0, $(i(t_0), \mu(t_0))$ belongs to this set. So for great-enough t_0 you can define $q(t_0)$ as giving the least q such that $i(q) \geq g^*$. The convergence of $i(t_0)$ to g^* implies that $q(t_0) - t_0$ converges to 0.

Next let $h(t_0)$ give the least h for which $\mu(h) = 1 - \omega(h)$ if that equality is anywhere satisfied and $q(t_0) + H$ for some large H otherwise. By (35) and the fact that $1 - \omega(t_0) \to \mu^*$ it must be that $h(t_0) - t_0$ is bounded above zero and hence that $h(t_0) > q(t_0)$ is ensured by choosing t_0 big enough.

For a big-enough t_0, then, $\mu(q(t_0)) < \mu^*$ from which (21) and (22) imply that for all $t \in (q(t_0), h(t_0))$, $i(t) > g^*$ and therefore

$$\breve{\mu}(t) = \frac{g^*}{\rho_G(t)} < \mu(t) = \frac{i(t)}{\rho_G(t)} < 1 - \omega(t). \tag{36}$$

But because $\mu(t) < 1 - \omega(t)$ for all t less than $h(t_0)$ you have also that $\rho_G(t) < \rho_R(t)$ in that interval and therefore that

$$\mu_R(t) \equiv \frac{g^*}{\rho_R(t)} < \breve{\mu}(t) < 1 - \omega(t). \tag{37}$$

Applying (27) to (36) gives that

$$\breve{\mu}(t)\chi(1 - \mu(t)) + (1 - \breve{\mu}(t))\gamma(1 - \mu(t)) > \breve{\mu}(t)\chi(\omega(t)) + (1 - \breve{\mu}(t))\gamma(\omega(t)). \tag{38}$$

while (37) implies that

$$\breve{\mu}(t)\chi(\omega(t)) + (1 - \breve{\mu}(t))\gamma(\omega(t)) > \mu_R(t)\chi(\omega(t)) + (1 - \mu_R(t))\gamma(\omega(t)). \tag{39}$$

Stacking these inequalities and noting that $1 - \breve{\mu}(t) < 1 - \mu_R(t)$ you get that

$$\frac{\breve{\mu}(t)}{1 - \breve{\mu}(t)}\chi(1 - \mu(t)) + \gamma(1 - \mu(t)) > \frac{\mu_R(t)}{1 - \mu_R(t)}\chi(\omega(t)) + \gamma(\omega(t)) \tag{40}$$

which is to say that $\hat{c}_G(t) > \hat{c}_R(t)$ for all t in $(q(t_0), h(t_0))$.

Let $\eta(t; t_0)$ give the ratio of sustainable socialist to capitalist consumption per worker at t given that the hypothetical change of regime occurs at t_0. Then you can write

$$\ln \eta(h(t_0) + t'; t_0) = \int_{t_0}^{q(t_0)} (\hat{c}_G(\tau) - \hat{c}_R(\tau))\, d\tau$$

$$+\int_{q(t_0)}^{h(t_0)} (\hat{c}_G(\tau) - \hat{c}_R(\tau))\, d\tau + \int_{h(t_0)}^{h(t_0)+t'} (\hat{c}_G(\tau) - \hat{c}_R(\tau))\, d\tau \tag{41}$$

As $t_0 \to \infty$ the first term approaches zero. By (40) the second term has a strictly positive limit. And because $c_R(t)$ approaches a path of exponential growth at the rate γ^*, the third term appoaches

$$\lim_{t_0 \to \infty} \int_{h(t_0)}^{h(t_0)+t'} \hat{c}_G(\tau)\, d\tau - \gamma^* t'$$

which is also greater than zero: As t_0 increases it's certain that $\breve{\mu}(t) \leq \mu(t) \leq \mu^*$ for any $t \geq t_0$, and section 6 showed that these inequalities imply $\hat{c}_G(t) \geq \gamma^*$. So the righthand side of (41) is strictly positive for great enough t_0 and for any t'.

As t goes to infinity, the ratio of the two systems' employment levels approaches $\left(\psi^{-1}[\gamma^*]\right)^{-1} \leq 1$, and rates of accumulation in both systems approach g^* so that actual and sustainable consumption coincide. Thus there are T_0 and T' such that the counterfactual formed from the capitalist economy at T_0 has greater per-capita consumption at all dates after $T \equiv T_0 + T'$.

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