Metroeconomica 56:1 (2005) 101–125

STEADY-STATE GROWTH AND DISTRIBUTION WITH AN ENDOGENOUS DIRECTION OF TECHNICAL CHANGE

A. J. Julius*
New School University

(June 2002; revised August 2003)

ABSTRACT

A model of labor-constrained accumulation and economically directed technical progress has a stable steady state at which the class distribution of income is invariant with respect to population and saving parameters yet sensitive to workers' stances in wage bargaining and to the tax and transfer policies of a redistributive state.

1. INTRODUCTION

The long-run profile of capitalist development includes a *labor-biased* pattern of technical change. Innovation has tended to decrease the quantities of labor necessary for the production of given quantities of goods, and there is no comparably general downward trend in requirements of produced nonlabor inputs. This pattern invites two kinds of explanation. On the one hand it is conceivable that a bias toward labor is already present in the flow of productively useful basic discoveries. Before they can show up in production, however, those discoveries must pass through a filter of economic decisions regulated by specific economic relations. So a second proposal to consider is that economic forces have selected the actual, labor-biased trajectory from a richer set of possible technological histories.

An old but long-submerged theme in growth economics, the hypothesis of an *endogenous direction* (ED) of technical change has resurfaced in a handful

* Conversations with Duncan Foley and time spent with his article and book (2003a, 2003b) got me onto this topic and shaped much of my work on it. I am grateful for his help. I also thank Per Gunnar Berglund, Anwar Shaikh, Lance Taylor and two referees of this journal for their criticisms of earlier drafts.

of recent discussions. In section 3 of this paper I encapsulate some old and new ED arguments in the following story. Suppose that firms produce goods of a single kind by hiring labor and tying up stocks of the good in proportions dictated by their knowledge at the time of production. And let them develop that knowledge along two dimensions, pursuing innovations that augment both labor and fixed capital at varying rates. The instantaneous change in the rate of profit on fixed capital due to innovation is a weighted sum of the rates at which innovation augments the two inputs: innovation's labor-augmenting component enters that average weighted by wages' current share of income, and capital augmentation by the profit share. Forty years ago Charles Kennedy pointed out that if firms choose the most rapidly cost-reducing innovations from a suitably structured set of possible technical changes, their choice is given by the profit share's ratio to the wage share. Gérard Duménil and Dominique Lévy have recently shown that wage and profit shares also pick out the mean direction of technical progress that emerges from an adaptive, stochastic counterpart to Kennedy's system. If firms draw labor- and capital-augmenting innovations from a stationary probability distribution, adopting only those innovations that promise to raise their profit rates, the time- and population-average profile of the resulting technical changes depends on income shares alone. It follows from either argument that a bias toward labor is possibly *induced* by the property relations of capitalist production. Where wages claim a sufficiently great share of income, they concentrate firms' innovation efforts on the augmentation of labor, imparting an upward trend to labor productivity even as the effectivity of nonlabor inputs stagnates or declines.

Duménil and Lévy (1995) have succeeded in recapitulating long time series of US technology and distribution with simulations that put this ED hypothesis to work. With their discoveries in mind I will assume here that ED is a promising explanation of long-run patterns of technical change, and I will not say anything to develop that promise. The purpose of this paper is to reconsider some familiar propositions of growth theory in its light. The strong form of labor bias known as *Harrod neutrality* is a necessary condition of the steady-state growth paths contemplated by most growth theory, and most growth theory has assumed such neutrality outright, confining technical change to the single dimension of pure labor augmentation. Growth theorists are not very proud of that restriction, so it is worth recalling that ED supports an alternative: Harrod-neutral progress can also emerge from firms' economically mediated exploration of a multidimensional technology. Steady-state growth and distribution have a different structure when they are grounded in this evolutionary neutralization of technical change, and I will discuss a few of the differences that it makes.

2. LABOR-LIMITED GROWTH AND ONE-DIMENSIONAL INNOVATION

To put down a benchmark, suppose first that a flow of automatically implemented discoveries augments labor's use in production at a constant proportional rate while changing nothing else. If the production of an economy's single good is represented by the Leontief technology in employed labor L and a non-depreciating stock of the good K,

$$f(L, K; t) \equiv \min[x(t)L, \rho(t)K] \tag{1}$$

then the assumption of exogenously Harrod-neutral technical change is that

$$\frac{\dot{x}}{x} \equiv \gamma > 0 \quad \text{and} \quad \frac{\dot{\rho}}{\rho} \equiv \chi = 0 \tag{2}$$

The effectivity of capital $\rho(t) = \rho(0) \equiv \bar{\rho}$ reads for the moment as a parameter of this accumulation process.[1]

This benchmark economy draws on an exogenous supply of labor L_s, which grows like

$$L_s(t) = \bar{L}_s \exp(nt) \tag{3}$$

What mechanisms might reconcile accumulation to labor's given path? Two of the best-known candidates involve labor markets in which wages signal the scarcity of labor. Defining the ratio of employed to available labor $l(t) \equiv L(t)/L_s(t)$, a law of motion for the real wage $w(t)$

$$\frac{\dot{w}}{w} = \psi(l) \qquad \psi' > 0 \text{ everywhere} \tag{4}$$

implies that the wage share of output, $\omega(t) = w(t)/x(t)$, obeys

$$\frac{\dot{\omega}}{\omega} = \psi(l) - \gamma \tag{5}$$

I assume that $0 < \psi^{-1}(\gamma) < 1$ for all admitted γ—income shares can always be put to rest by some suitably massive reserve army of jobless workers.

[1] The entire paper carries forward this paragraph's one-good abstraction, and I do not claim that its arguments are true of real economies with their multiplicities of produced inputs.

If capitalists are the only savers and if they save from their profits in the proportion s, saving accounts for a share $s(1 - \omega)$ of social income. Assume that firms' investment outlays are adjusted so that the stock of fixed capital is continuously used at full capacity as it expands at the corresponding warranted rate

$$\frac{\dot{K}(t)}{K(t)} \equiv g_k(t) = s[1-\omega(t)]\overline{\rho} \tag{6}$$

From (2), (3) and (6), the employment ratio l then obeys

$$\frac{\dot{l}}{l} = s(1-\omega)\overline{\rho} - \gamma - n \tag{7}$$

Equations (5) and (7) are a complete dynamical system in ω and l, basically Goodwin's predator–prey model of cyclical growth.[2] Away from the origin this system has one rest point

$$l^* = \psi^{-1}(\gamma) \tag{8}$$

and

$$\omega^* = 1 - \frac{\gamma + n}{s\overline{\rho}} \tag{9}$$

where the wage share sends capital accumulation up a path parallel to effective labor supply. That critical wage share is lower, the more rapid the reproduction of the workforce or the more profligate capitalists' consumption of their profits, and the exogenous augmentation of labor in technical change appears as a kind of *ersatz* population growth, with

$$\frac{\partial\omega^*}{\partial\gamma} = \frac{\partial\omega^*}{\partial n} < 0 \tag{10}$$

A second possible mechanism is that of Solow and Swan. Let the Leontief technology give way to a smooth, concave production function, $F[x(t)L,$

[2] Goodwin (1967) emphasized that his model's paths are closed orbits through their initial conditions; as later writers have repeatedly remarked, these cycles are not robust to "small" respecifications of that model.

$\rho(t)K$], that is homogeneous of the first degree. Suppose again that technical change is purely labor-augmenting in the sense of (2). Then output per effective employed worker can be written

$$f[k(t)] \equiv F[1, k, (t)] \tag{11}$$

where

$$k(t) \equiv \frac{\bar{\rho}K(t)}{x(t)L(t)}$$

Constant social saving in the proportion s gives rise to a differential equation for the full-employment capital to effective labor ratio

$$\dot{k} = s\bar{\rho}f(k) - (\gamma + n)k \tag{12}$$

for which the restrictions on f ensure a stable critical point k^* with

$$\frac{\partial k^*}{\partial n} < 0 \qquad \frac{\partial k^*}{\partial \gamma} < 0 \qquad \frac{\partial k^*}{\partial s} > 0 \tag{13}$$

A unique value of the wage rate of effective labor, $v(t) \equiv w(t)/x(t)$, induces profit-maximizing firms to employ the entire predetermined workforce on the predetermined capital stock; by the concavity of F, this market-clearing effective wage is increasing in k. From (13) it follows that

$$\frac{\partial v^*}{\partial n} < 0 \qquad \frac{\partial v^*}{\partial \gamma} < 0 \qquad \frac{\partial v^*}{\partial s} > 0 \tag{14}$$

Comparative dynamics of the wage *share* satisfy the same pattern of inequalities just in case f's elasticity of substitution is bounded between 0 and 1.

While they assign distinct roles to wage motions and saving and production decisions as regulators of accumulation, these Goodwin and Solow–Swan arguments imply in common that the steady-state income distribution is uniquely selected by parameters of labor supply and saving. Should workers try to raise their wages through collective action or public policy, their efforts are certain to fail in this labor-limited long run. However these exacting conclusions will not survive the generalization to a second dimension of technical change.

3. ECONOMICALLY DIRECTED INNOVATION

This section allows technical progress to augment fixed capital and subjects its direction to economic influence. Suppose first that labor- and capital-augmenting innovations are the predictable outcomes of research activities undertaken by capitalist firms.[3] Every combination of these activities results in a definite profile of technical change, represented by a couple (χ,γ), and the set of feasible (χ,γ) pairs has as its boundary Kennedy's *innovation possibilities frontier*; there is a function g such that for every feasible (χ,γ)

$$\gamma \leq g(\chi) \qquad g' < 0, g'' < 0 \tag{15}$$

A firm that expects to face a rate of real wage growth χ on an aggregate labor market where its own actions are vanishingly small can maximize the instantaneous proportional rate of reduction in its unit production cost c by choosing

$$\begin{aligned} &(\chi,\gamma) \text{ to maximize} -\frac{\dot{c}}{c} = \omega(\gamma-\psi)+(1-\omega)\chi \\ &\text{subject to } \gamma \leq g(\chi) \end{aligned} \tag{16}$$

The stated shape of $g(\chi)$ ensures that the firm's reduced first-order condition for this problem

$$-g'(\chi) = \frac{1-\omega}{\omega} \tag{17}$$

allows its chosen rates of capital and labor augmentation to be expressed as functions of the current wage share:

$$\chi(t) = \chi[\omega(t)], \qquad \gamma(t) = \gamma[\omega(t)], \qquad \text{with } \gamma' > 0 > \chi' \tag{18}$$

From the perspective of general production sets, the stipulation that innovation augments inputs still looks quite 'special', notwithstanding this breakthrough to a second input. Also it is implausible that firms face the same innovation possibilities for all time and in every state of the technology. And if the innovation frontier changes position as time or progress unfolds, any

[3] Rooted in Hicks (1932, ch. 6), the mechanism of this paragraph has its immediate sources in Kennedy (1964), von Weizsäcker (1966) and Samuelson (1965). For recent microfounding reconstructions, see Acemoglu (2001) or Funk (2002).

hope of steady-state analysis is seemingly stillborn.[4] Of course these troubling objections are no reason to prefer the assumption that a rate and direction of technical progress are given to the economy independently of time, the state of the art and the state of everything else.

Having granted a stationary set of input-augmenting innovation possibilities, we may still doubt that firms would know which set they face so that they might carry out the program (16). Duménil and Lévy escape this second objection by rebuilding the induced bias story on adaptive foundations.[5] Suppose now that each member of a population of identical firms draws innovations from a stationary distribution on the innovation set, again some subset of the χ, γ plane containing (0, 0) as an interior point. In the spirit of Okishio (1961) let each firm adopt only those innovations that increase the instantaneous change in the rate of profit on its fixed capital for an expectation of real-wage growth ψ that the firm takes as independent of its own activity. Differentiating through the profit rate $r \equiv (1 - \omega)\rho$ with respect to time and expressing its time derivative as a function of the going rates of labor augmentation, capital augmentation and real-wage growth,

$$\dot{r}(\chi, \gamma, \psi) \equiv \rho[(1-\omega)\chi + \omega(\gamma - \psi)] \tag{19}$$

a generalized Okishio rule says to implement an innovation (χ, γ) if and only if

$$\dot{r}(\chi, \gamma, \psi) \geq \dot{r}(0, 0, \psi) \tag{20}$$

This rule implies that the change goes through if and only if

$$\rho[(1-\omega)\chi + \omega\gamma] \geq 0 \tag{21}$$

The set of *viable* innovations—those that permeate the filter (20)—is bounded from below by a line

$$\gamma = -\left(\frac{1-\omega}{\omega}\right)\chi \tag{22}$$

[4] These objections are in Samuelson (1965), Nordhaus (1973) and Arrow (1969); Skott (1981) studies an ED model with a nonstationary innovation frontier.
[5] See Duménil and Lévy (1995, 2003); apart from their deterministic ancestors of the 1960s, the stochastic set-up of those papers is anticipated by Nelson and Winter's (1982, ch. 9) evolutionary arguments.

which recalls the firm's first-order condition (17) in the deterministic Kennedy set-up. For a given distribution on the innovation set, the mean adopted profile of technical change, equal to the center of gravity of the set of viable innovations, depends only on the profit share's current ratio to the wage share. Average labor augmentation is increasing and average capital augmentation decreasing in the wage share, as can seen by supposing that χ has the horizontal axis in χ, γ space: a rise in the wage share rotates the viability line (22) counterclockwise; the viable subset loses weight in its southeast, capital-augmenting, labor-disaugmenting region, and gains weight in its northwest, labor-augmenting, capital-disaugmenting region; and its center of gravity shifts to become more labor-augmenting and less capital-augmenting than at the initial wage share.

In the rest of this paper I assume that an aggregate profile of technical change is described by respectively decreasing and increasing differentiable functions of the wage share $\chi(\omega)$ and $\gamma(\omega)$, and I distinguish between Kennedy's derivation and a deterministic approximation of Duménil and Lévy's model—in whose case those functions name the mean profile of technical change picked out by the wage share for some distribution on the innovation set—only where the choice of derivation matters to the argument at hand.[6]

4. GROWTH AND DISTRIBUTION WITH A VARIABLE BIAS OF TECHNICAL CHANGE

I will now embed this technical change mechanism in the Goodwin growth model of (5) and (7), acquiring $\rho(t)$ as a state variable of the three-dimensional flow

$$\dot{\omega} = [\psi(l) - \gamma(\omega)]\omega \tag{23}$$

$$\dot{l} = [s(1-\omega)\rho + \chi(\omega) - \gamma(\omega) - n]l \tag{24}$$

$$\dot{\rho} = \chi(\omega)\rho \tag{25}$$

[6] In arguing for these functions I have helped myself to a representative capitalist and to hand-waving deterministic approximation. But in Julius (2001) large populations of capitalists are bombarded with randomly arriving innovations which they adopt or reject according to decision rules and individual information that coevolve with the distribution of technologies and wages. Simulations recover the main macroscopic features of the steady states that I discuss in sections 4 and 6.

The equations describing a nonzero rest point of this system decompose in an economically intriguing way.[7]

Consider first the requirement that capital augmentation go to zero. Where section 2 simply stipulated that neutrality, the new system evolves so as to satisfy it. In Kennedy's model, if the innovation possibilities frontier has $g'(0) < 0$, there exists a unique value of the wage share

$$\omega^* = \chi^{-1}(0) \tag{26}$$

that directs firms to aim for $(0, g(0))$ as the constrained-best innovation according to their problem (16); at that wage share, capital augmentation is turned off. A steady-state wage share is likewise distinguished for any of a large class of technical change distributions in Duménil and Lévy's framework as that value which sends the center of gravity of the viable set, bounded by (22), to the Harrod-neutral locus $\chi = 0$. In the remainder of the paper I will assume that (26) has a solution ω^* between 0 and 1, which at once picks out a steady-state rate of labor augmentation $\gamma(\omega^*)$. Income distribution and productivity growth are pinned down by the neutrality requirement, and the population growth rate and capitalists' saving propensity are given from outside this system, so it falls to the effectivity of fixed capital to hold constant the employment ratio. Its steady-state value emerges through (24) as

$$\rho^* = \frac{\gamma(\omega^*) + n}{s(1-\omega^*)} \tag{27}$$

while the wage-share-stabilizing employment ratio is now picked out from (25) as

$$l^* = \psi^{-1}[\gamma(\omega^*)] \tag{28}$$

Steady-state distribution is independent of the rates of population growth and capitalist saving that govern the productive availability of labor and

[7] I owe my understanding of this model to Duncan Foley (2003a, 2003b). Shah and Desai (1981) were apparently the first to consider this system; their focus was on the robustness of Goodwin's cycles, a theme also taken up by van der Ploeg (1987). Thompson (1995) has argued keenly for the stabilizing role of wage/employment feedback in two-dimensional technical evolution. With its mixture of reserve-army wage dynamics, profitability-dependent accumulation and endogenous mechanization, (23)–(25) might also ring a bell for some readers of volume 1, chapter 25, of Marx's *Capital*.

capital goods, and the endogenous effectivity of capital takes over from distribution the burden of harmonizing capital accumulation and the growth of population, with $\partial\rho^*/\partial n > 0$ and $\partial\rho^*/\partial s < 0$.

An appendix shows that this steady state is locally stable if the technical change functions are derived in Kennedy's framework. As Shah and Desai discovered, Goodwin's neutral oscillations are overcome by the superimposed forces of endogenous technical change. So there is good reason to focus on steady states as long-run attractors. Before resuming that focus, however, I should mention an alternative that is worth investigating. Because the Duménil–Lévy variant of an ED model places fewer restrictions on the technical change functions, it fails to rule out an unstable rest point for (23)–(25). The appendix shows that if instability occurs anywhere in the parameter space, it arises via a Hopf bifurcation. This fact implies that limit cycles are a second dynamical possibility, and my simulations have confirmed that, in the appropriate parametric region, distribution, accumulation and technology are subject to persistent fluctuations.[8]

Returning to the steady-state properties of ED-based growth, notice that these are reproduced when ED is exported to a Solow–Swan economy with synchronic substitution and continuous full employment. The differential equation (12) for the effective capital to effective labor ratio becomes

$$\dot{k} = s\rho f(k) + \{\chi[\omega(k)] - \gamma[\omega(k)] + n\}k \tag{29}$$

with ρ a second state variable governed by

$$\dot{\rho} = \chi[\omega(k)]\rho \tag{30}$$

and with the function $\omega(k)$ defined as giving the wage share consistent with profit maximization and labor-market clearing at k. I assume an elasticity of subsitution bounded above zero and below one, so that $\omega'(k) > 0$ everywhere.[9] A steady-state wage share again emerges prior to the accumulation mechanism in the requirement of Harrod neutrality,

$$\omega(k^*) = \chi^{-1}(0) \tag{31}$$

---

[8] Nor do periodic motions exhaust the possibilities. By weakly coupling three or more regional or national Goodwin/ED systems like (23)–(25), I have been able to generate strange attractors along the "Ruelle–Takens route to chaos" that Lorenz (1987) and Lordon (1995) have explored in related economies.

[9] That assumption on f is also necessary and sufficient for the steady state's stability; see Drandrakis and Phelps (1966) for this and other details of the neoclassical dynamics.

Since $\omega'(k) > 0$, (31) solves uniquely for k^* as

$$k^* = \omega^{-1}[\chi^{-1}(0)] \tag{32}$$

determining the steady-state wage of effective labor v^* as the value that qualifies this k^* as profit-maximizing. In a departure from the conclusions (13) and (14) reached in the case of exogenous technical change, the long-run capital intensity, effective wage and interest rate are all invariant with respect to thrift and population growth, while $\dot{k}$ is sent to zero by a value of the endogenous capital effectivity

$$\rho^* = \frac{\gamma[\omega(k^*)] + n}{sf(k^*)} \tag{33}$$

in a manner akin to its regulation of Goodwinian accumulation at (27).

Where production is constrained by exogenous labor and labor's scarcity is signalled by the short-run motions of real wages, technical change degenerates to Harrod neutrality as distribution enters a constant configuration bearing no trace of labor supply or saving parameters. To underline the role that a wage-mediated labor constraint plays in shaping this evolution, it might help to withdraw that constraint temporarily. The next section considers two-dimensional innovation in a model of abundant labor and exogenous wages.

5. CONVENTIONAL WAGES AND DIRECTED TECHNICAL PROGRESS

Now suppose that labor is elastically supplied at a real wage whose path is determined independently of the accumulation process by some complex of material culture, social convention, state policy or institutionally specific bargaining. Absent technical progress, this thought suggests an exogenously constant real wage. But if labor's productivity is increasing forever through technical change, a constant real wage implies that the share of wages approaches zero as time goes to infinity. To preclude that bourgeois utopia some writers have instead assumed that wages keep pace with productivity so as to maintain an exogenous wage *share*. If the share of saving in national income is again the Cambridge proportion $s(1 - \omega)$, then a conventional wage share $\overline{\omega}$ stabilizes accumulation at the rate

$$g_k^* = s(1 - \overline{\omega})\overline{\rho} \tag{34}$$

Now suppose that innovation is directed by the ED mechanism of section 3. The effectivity of capital ρ is a state variable evolving subject to (25), and a constant wage share implies that ρ is sent down an exponential path like

$$\rho(t) = \rho_0 \exp(\chi(\overline{\omega})t) \tag{35}$$

Capital accumulation is

$$g_k(t) = s(1-\overline{\omega})\rho_0 \exp(\chi(\overline{\omega})t) \tag{36}$$

and it explodes or dies out according as the conventional wage share stands below or above the value that directs the innovating capitalists to a Harrod-neutral path of progress.

Similar reasoning applies to economies in which real wages grow at a proportional rate ψ independent of the path of productivity. The wage share then follows

$$\dot{\omega} = [\psi - \gamma(\omega)]\omega \tag{37}$$

and its rest point is some $\omega^{**} = \gamma^{-1}(\Psi)$. This is stable since $\gamma' > 0$; ω^{**} attracts all paths of the wage share, which is the only state variable in its own law of motion, whatever accumulation dynamic is adduced. This attracting wage share again generally differs from the value $\omega^* = \chi^{-1}(0)$ that turns off innovation's augmentation of capital. Growth rates tend to zero if $\gamma^{-1}(\psi)$ is greater than $\chi^{-1}(0)$ and to infinity if it is less. Notice that for a *constant* conventional wage, $\psi = 0$, the technical change mechanism implies that

$$\omega^{**} = \gamma^{-1}(0) < \chi^{-1}(0) = \omega^* \tag{38}$$

and therefore that

$$\chi(\omega^{**}) > 0 \tag{39}$$

Capitalist growth is necessarily explosive, and it calls for an accommodating explosion of labor supply.

Finally suppose with Thomas Michl (1999) that instead of passing productivity growth entirely through to wage growth, labor-market institutions permit only a partial passthrough, represented by a rule like

$$w(t) = x(t)^{\mu} \qquad \mu \in (0,1] \tag{40}$$

from which

$$\dot{\omega} = (\mu - 1)\gamma(\omega)\omega \tag{41}$$

gives the evolution of wages' share.[10] Equation (41) has the rest point $\omega^{***} = \gamma^{-1}(0)$, which for $\mu < 1$ is globally stable, again because $\gamma' > 0$. So the conclusion of (39) encompasses ω^{***} also; capital augmentation is perpetually positive and causes accumulation to diverge.

Where wages evolve independently of accumulation, ED appears to collapse the possibility of nonexplosive long-run growth to a parametric fluke. Only some additional nonlinearity or nonstationarity—for example the inward displacement of the innovation frontier considered by Duménil and Lévy (2003)—can contain the unstable trajectories that result.

According to one familiar map, rival growth theories are best individuated by the labor-market and saving/investment assumptions with which they close a common model of production. These alternative 'closures' support alternative sets of steady-state comparative dynamics that are supposed to represent the bedrock explanatory differences between theories.[11] The best-known closures share a presupposition of exogenously Harrod-neutral technical progress, and I have argued that to replace that restriction with economically directed innovation is to change their signature long-run behavior. The distribution-determined steady states of the labor-surplus economy, for example, are destroyed in this revision. And in labor-limited systems steady-state income distribution ceases to index scarcity, becoming instead insensitive to parameters of saving or labor supply. Technical change looms large on the sorts of time scales for which steady-state comparisons are possibly informative. So we need to keep in mind that the pattern of those comparisons depends on how we have chosen to account for the *direction* of technical change.

The remainder of this paper resumes a reconsideration of labor-constrained accumulation from the ED point of view. I ask whether redistributive policy or collective action can alter the class distribution of income and wealth in steady-state growth. The question is a heuristic for exploring ED-based steady states, but I think that it also has some intrinsic interest. Under the exogenously neutral technical change of section 2 the answer was "no", but ED reopens the question.

---

[10] Michl studies bargaining like (40) on the assumption that capitalists apply the Okishio filter to innovations drawn from an exogenously labor-saving, capital-using flow. Innovation continues until the wage share has declined to a value such that innovation is no longer profitable. The text reaches a different conclusion by relaxing Michl's restriction of technical change to a point in the χ, γ plane.

[11] For a canonical statement of this outlook see Marglin (1984).

6. TECHNICAL CHANGE INTERNALIZED IN WAGE BARGAINING

I begin with a slightly closer look at wage bargains like (40). Suppose that while a fraction of labor productivity growth is passed through to the growth of real wages, wages also respond to the labor market's tightness. Taking the suggestion of van der Ploeg (1987), I rewrite the wage share's law of motion as

$$\dot{\omega} = [\psi(l) + (\mu - 1)\gamma(\omega)]\omega \tag{42}$$

where $\mu \in (0, 1]$ is an elasticity of the wage with respect to labor productivity as in (40) above; the employment ratio l again follows the Goodwin motion (24) and ρ's evolution is governed by (25). Where the employment ratio takes the value

$$l^* = \psi^{-1}\{(1 - \mu)\gamma[\chi^{-1}(0)]\} \tag{43}$$

real wages are increasing parallel to the path of labor productivity selected by the Harrod-neutral outcome of the ED mechanism. Productivity passthrough in the wage bargain leaves long-run distribution untouched, and it requires offsetting unemployment, since (43) implies $\partial l^*/\partial\mu < 0$.

The previous paragraph has a hidden premise that needs to be rethought. I have argued as though the interactions relating wages to production technology were strictly external to the firm. And a small firm can certainly afford to overlook the impact of its own innovation on aggregate labor demand. But innovation has the second effect of repositioning the wage–profit frontier that governs the firm's bargaining with workers who will operate the adopted technology. This second effect arguably *should* show up in the firm's innovation decision.

Though this observation deserves to be spelled out in a fullblown model of ongoing bargaining and multidimensional innovation—one along the lines of Skillman (1997), for example—a simple adaptation of (42) suffices to make my point about the steady–state set and is appropriate given Michl's and van der Ploeg's discussions of technical change in closely related bargaining regimes.[12] Let each firm choose an innovation target (χ,γ) to maxi-

[12] If workers for their part anticipate the innovation decisions called forth by their wage demands, and if they see a high probability of long tenure in their current jobs, they might opt for restraint to induce a higher path of firm-level productivity. Better still they might look for a commitment device—militancy, anyone?—that allows them to set *high* wage demands *independent* of productivity. The myopia assumed by the text might represent workers who expect to move soon to other jobs or unemployment.

mize the resulting instantaneous increase in the profit rate on its capital stock, subject to the Kennedy innovation frontier and to a wage bargain that the firm believes will follow (42). Differentiating through the profit rate with respect to time and substituting for $\dot{\omega}$. from (42) gives

$$\dot{r} = \rho\{(1-\omega)\chi + \omega[\psi(l) + (1-\mu)\gamma]\} \tag{44}$$

The choice of (χ, γ), to maximize this gain in profitability subject to $\gamma \leq g(\chi)$ must have

$$-g'(\chi) = \frac{1-\omega}{\omega(1-\mu)} \tag{45}$$

which specifies (χ, γ) implicitly as a function of μ given the parameter μ and provided μ is less than one; a more intense passthrough prompts the firm to augment labor less.[13]

If productivity growth is entirely passed through, the firm's best innovation target is undefined in (45). Innovation's labor-augmenting component drops out of (44), and the firm maximizes the change in its profit rate by choosing the highest rate of capital augmentation available on the innovation frontier, a tendency that does not sit well with the stylized facts of capitalist technical progress.

But assume that μ is less than 1, so that a steady state exists. The employment ratio there continues to show $\partial l^*/\partial\mu < 0$. But the wage share that satisfies (45) with Harrod neutrality is

$$\omega^* = \frac{1}{1-g'(0)(1-\mu)} \tag{46}$$

which goes to 1 as μ goes to 1. The diminished payoff to labor augmentation entailed by a more intense passthrough needs to be offset by a greater weight for wages in costs if firms are to favor a neutral direction of innovation.[14]

[13] A referee for this journal points out that if labor- and capital-augmenting innovation are symmetrically passed through to wages, then the intensity of passthrough makes no difference to a firm's direction of innovation. My conclusion requires only that wages be *more* elastic with respect to the labor component.

[14] An appendix shows that this steady state is locally stable and explains how this conclusion is consistent with van der Ploeg's (1987) finding that instability is possible in an apparently equivalent model.

In drawing attention to this fact, I do not claim that workers do *well* to stake out a high μ. Apart from its employment effects, an intense passthrough, by retarding a firm's productivity growth in the transition to its Harrod-neutral phase, might deliver permanently lower wages than can be had along paths with lower μ. I have waded into these issues only far enough to demonstrate that if μ indexes workers' institutionally variable capacity for pressing their wage demands, that capacity shows up in the steady-state income distribution provided that firms take account of it as they choose a direction for their technical changes.

7. A PROFITS TAX

Endogenously directed innovation entails as well a collective fiscal capacity for cross-class redistribution in steady growth. Suppose that a government decides to tax firms' profits at a rate τ and to transfer its revenue from the tax to workers as a social wage. Where available labor grows at the rate n, technical change augments labor at the rate γ, capital effectivity is locked into $\bar{\rho}$ and capitalists do all the society's saving in the constant proportion s, the steady-state profit rate r and net-of-taxes profit share

$$\pi^{\mathrm{N}} \equiv (1-\omega^*)(1-\tau) \tag{47}$$

are wholly insulated from taxation, since they must satisfy

$$r^* = \bar{\rho}\pi^{\mathrm{N}} = \frac{n+\gamma}{s} \tag{48}$$

for steady growth. A social wage financed by this tax can only supplant private wages, and labor's share net of transfers

$$\omega^{\mathrm{N}} \equiv \omega^* + \tau(1-\omega^*) = 1-\pi^{\mathrm{N}} = 1-\frac{n+\gamma}{s\bar{\rho}} \tag{49}$$

is beyond the reach of the scheme.[15]

But suppose that labor- and capital-augmenting innovation is again directed by firms to maximize the instantaneous gain in the rate of profit on

[15] See Steedman (1973) for a technologically generalized discussion of long-run barriers to redistributive taxation in labor-constrained growth.

fixed capital subject to Kennedy's innovation frontier $\gamma \leq g(\chi)$. Then although the steady-state profit rate is still impervious to the tax,

$$r^* = \rho^* \pi^{\mathrm{N}} = \frac{n + g(0)}{s} \tag{50}$$

comparative dynamics of the profit *share* are a different story. For Harrod neutrality wages' share satisfies

$$\omega^* = \frac{1}{1 - g'(0)} \tag{51}$$

implying a net profit share

$$\pi^{\mathrm{N}} = (1 - \omega^*)(1 - \tau) = \frac{-g'(0)(1 - \tau)}{1 - g'(0)} \tag{52}$$

while an endogenous steady-state effectivity of capital takes on the value that solves (50), or

$$\rho^* = \frac{n + g(0)}{s\pi^{\mathrm{N}}} = -\frac{[n + g(0)][1 - g'(0)]}{sg'(0)(1 - \tau)} \tag{53}$$

Private wages and profits share the burden of profits taxation in proportions given by the slope of the innovation frontier at the point of Harrod-neutral innovation. And if the entire tax revenue redounds as social wages to the working class, this scheme selects a steady-state net labor share equal to

$$\omega^{\mathrm{N}} = \frac{1 - \tau g'(0)}{1 - g'(0)} \tag{54}$$

which has $\partial\omega^{\mathrm{N}}/\partial^*\tau > 0$.

8. PASINETTI REPROCESSED

In a final attempt at defamiliarization this section revisits the 1960s discussion of so-called Pasinetti/anti-Pasinetti theorems from the standpoint of endogenously directed technical change. Such analysis takes explicit account of the wealth distribution that evolves where workers are assumed to save.

To sharpen the issues, I will consider only the simplest environment in which they arise. Suppose that working-class households purchase claims to productive assets and take dividends on these at the same rate of profit r as is faced by purely capitalist, non-wage-earning households or capitalist firms. Let s_c be the constant ratio of those capitalists' savings to the profits that exhaust their income, and let the workers save out of their wages and property income in a strictly lower constant proportion s_w. Suppose that technical change is labor-augmenting at the rate γ. If both classes are indeed to hold property in the long run, a steady state requires that the stock owned by each class expand at the same rate as the total effective workforce, $x(t)L_s(t) = \bar{x}\bar{L}_s \exp(n + \gamma)t$. We need

$$s_c r - \gamma - n = 0 \tag{55}$$

for constant capitalist wealth per effective worker; and, for unchanged average effective-worker holdings,

$$\frac{\dot{K}_w}{K_w} - n - \gamma = \frac{s_w(rK_w + \omega\rho K)}{K_w} - n - \gamma = s_w\left(r + \frac{\omega\rho}{\kappa}\right) - n - \gamma = 0 \tag{56}$$

where K and K_w are the total capital stock and the stock claimed by workers and κ is the workers' share of wealth K_w/K. Pasinetti (1962) pointed out that this two-class steady state decomposes: the first of these equations alone determines a long-run equilibrium profit rate

$$r^* = \frac{n + \gamma}{s_c} \tag{57}$$

whose value is independent of workers' saving decisions.

If output's ratio to capital is technologically fixed at $\bar{\rho}$, (57) immediately selects the steady-state wage share

$$\omega^* = 1 - \frac{n + \gamma}{s_c \bar{\rho}} \tag{58}$$

Substitution for r and ω from (57) and (58) into (56) implies that the workers' share of propertied wealth in a two-class equilibrium is

$$\kappa^* = \frac{s_w(s_c\bar{\rho} - n - \gamma)}{(s_c - s_w)(n + \gamma)} \tag{59}$$

But that equilibrium need not exist. From (59) it is clear that $0 < \kappa^* < 1$ if and only if

$$s_c > \frac{n+\gamma}{\bar{\rho}} > s_w \tag{60}$$

and where the second inequality is unsatisfied pure capitalists are excluded from any steady state. But then there is no steady state unless $s_w \bar{\rho}$ happens to *equal* $n + \gamma$. On the other hand if the technology allows for immediate substitution among production activities, and if the function $q(r)$ gives the output–capital ratio corresponding to the activity that minimizes costs at r, the inequality (60) becomes

$$s_c > \frac{n+\gamma}{q[(n+\gamma)/s_c]} > s_w \tag{61}$$

Samuelson and Modigliani (1966) noticed that if the second inequality fails there can exist a second 'anti-Pasinetti' steady state, ridden of pure capitalists, with a profit rate r^{**} such that firms operate activities giving an output–capital ratio

$$q(r^{**}) = \frac{n+\gamma}{s_w} \tag{62}$$

The existence condition (61) has attracted a lot of mathematical name-calling—is it the 'general' or the 'special' case?

ED makes it possible to sidestep such disputes. Consider a two-class wealth dynamics with an endogenous direction of technical change. If steady growth is to sustain positive purely capitalist wealth, an equilibrium profit rate again falls out of an equation like (55) alone. Even barring any direct substitution among previously discovered techniques, however, ρ is now a *variable* in this two-class equilibrium condition:

$$s_c(1-\omega)\rho - \gamma(\omega) - n = 0 \tag{63}$$

And the steady-state wage share is determined independently of (63) in the requirement (26) that capital augmentation be turned off. So (63) solves, not for an equilibrium income distribution, but for an equilibrium capital effectivity

$$\rho^* = \frac{n + \gamma[\chi^{-1}(0)]}{s_c[1 - \chi^{-1}(0)]} \tag{64}$$

Substitution from (26) and (64) into the second Pasinetti condition (56) implies that, if a class-divided steady state exists, the wealth distribution there is

$$\kappa^* = \frac{s_w \chi^{-1}(0)}{(s_c - s_w)[1 - \chi^{-1}(0)]} \tag{65}$$

A necessary and sufficient condition for capitalist survival is therefore that

$$\frac{s_c - s_w}{s_w} > \frac{\chi^{-1}(0)}{1 - \chi^{-1}(0)}$$

or

$$s_c(1 - \omega^*) > s_w \tag{66}$$

Where (66) fails, purely capitalist weath holding is anathema to steady-state growth. The only equilibrium in such a case has $\kappa^* = 1$, from which (56) becomes

$$s_w(r + \omega\rho) - n - \gamma[\chi^{-1}(0)] = s_w\rho - n - \gamma[\chi^{-1}(0)] = 0 \tag{67}$$

identifying the long-run effectivity of capital as

$$\rho^{**} = \frac{n + \gamma[\chi^{-1}(0)]}{s_w} \tag{68}$$

Technological endogeneity has a disorienting upshot for these wealth dynamics. That it cuts across the earlier lines of discussion is brought out by comparing the inequalities (60), (61) and (66). Each states a necessary and sufficient condition for two-class long-run equilibrium. In all cases workers cannot be too thrifty nor capitalists too eager to consume. But where a failure of (60) implies that no steady state exists, the alternative to a satisfied (66) is the anti-Pasinetti equilibrium, secured by capitalists' sequential choices of techniques in the ED process rather than by the synchronic substitution of (62). For a given population growth rate and workers' saving propensity, (60) requires that the exogenous output–capital ratio not be too high, and some more complex condition on production techniques is embedded in the inequality (61). But for given saving propensities (66) only bounds from above the neutralizing value of the wage share. And where (60) and (61) define a region of population and productivity growth rates consistent with two-class equilibrium, (66) makes no mention of those parameters; the class-divided steady state does not depend for its possibility on contingencies of labor supply.

I think there is some political economy in these comparisons. The parameters related by the existence conditions (60) or (61) are all beyond the reach of obvious collective action, save for the possibility that a socialization of workers' saving through pension or employee-buyout schemes could raise its overall rate. So though an unsatisfied (61) describes circumstances under which an economy might blindly converge to the anti-Pasinetti outcome of no pure capitalists, it does not tell people how to aim for that result except by getting a collective grip on their saving. By contrast (66) implies that, for a given ratio of capitalist to worker saving propensities, some sufficiently high steady-state wage share suffices to rule out the two-class destination. But then recall from section 6 that, if wage bargains internalize ongoing technical progress, the neutralizing wage share is higher, the more intense the passthrough of productivity gains to wage growth. Upon substitution for the wage share from (46), (66) implies that if

$$\mu > 1 - [-g'(0)(s_c/s_w - 1)]^{-1} \tag{69}$$

nonworkers can own no capital in a steady state. If the classless equilibrium is stable in the case where no class-divided equilibrium exists—the simulations that I have looked at bear this out—it follows that to achieve an eventual euthanasia of the *rentier* workers need only organize for higher wages from day to day.

9. CONCLUSION

If wages have enough weight in capitalists' costs, they will skew capitalists' multidimensional innovation efforts toward a rapid reduction of labor requirements at the expense of other improvement. This paper has assembled a model of mutually conditioned growth and productive evolution whose orbits converge to such a labor-biased pattern from arbitrary starting points. The degeneration of technical progress toward Harrod neutrality along this model's trajectories reflects the asymmetry between capital and labor inputs—a distinction between directly produced commodities whose augmentation by technical progress would allow them to reproduce themselves at an ever-accelerating rate and human capacities whose formation is subject to institutional and biological inertia—and requires that wages signal the pressure of accumulation on the system's reserves of labor. The power of this endogenous explanation of labor bias and its appeal to labor as a constraint on production encourage us to re-examine the character of labor-limited growth. Where technology is confined to a given effectivity of capital, the wage effects of bargaining institutions or tax policy die out as distribution approaches a steady state; where productive change has an economically variable direction, bargaining or taxes persist in shaping the attracting distributions of income and wealth.

APPENDIX: STABILITY ANALYSIS OF THE STEADY STATES OF SECTIONS 4 AND 6

From section 4: endogenous technical change with scarce labor and classical accumulation

The section describes a three-dimensional dynamical system,

$$\dot{\omega} = [\psi(l) - \gamma(\omega)]\omega \tag{A1}$$

$$\dot{l} = [s(1-\omega)\rho + \chi(\omega) - \gamma(\omega) - n]l \tag{A2}$$

$$\dot{\rho} = \chi(\omega)\rho \tag{A3}$$

Linearizing around the nonzero point of rest gives the Jacobian

$$J = \begin{pmatrix} -\omega^* \gamma'(\omega^*) & \omega^* \psi'(l^*) & 0 \\ l^* [-s\rho^* + \chi'(\omega^*) - \gamma'(\omega^*) & 0 & s(1-\omega^*)l^*] \\ \rho^* \chi'(\omega^*) & 0 & 0 \end{pmatrix} \tag{A4}$$

The characteristic equation of this matrix has the form

$$\lambda^3 - \text{tr}J \cdot \lambda^2 + \text{pm}J \cdot \lambda - |J| = 0 \tag{A5}$$

with λ an eigenvalue of the matrix, tr$J$ its trace, $|J|$ its determinant and pm J the sum of its second-order principal minors. The Routh–Hurwitz necessary and sufficient conditions for eigenvalues with uniformly negative real parts require that the Jacobian's trace, $-\omega^*\gamma'(\omega^*)$, and its determinant,

$\omega^*\psi'(l^*)s(1-\omega^*)l^*\rho^*\chi'(\omega^*)$, both be less than zero, which is true for all admissible parameter values. Those conditions require also that

$$\text{pm}J = l^*[s\rho^* - \chi'(\omega^*) + \gamma'(\omega^*)]\omega^*\psi'(l^*) > 0 \quad \text{(A6)}$$

which holds necessarily, and that

$$-\text{pm}J + \frac{|J|}{\text{tr}J} < 0 \quad \text{(A7)}$$

which is true if and only if

$$H \equiv -[n+\gamma(\omega^*)]\left[\frac{1}{1-\omega^*} + \frac{\chi'(\omega^*)}{\omega^*\gamma'(\omega^*)}\right] + \chi'(\omega^*) - \gamma'(\omega^*) < 0 \quad \text{(A8)}$$

This inequality is satisfied given Kennedy's optimizing derivation of the technical change functions: the firm's first-order condition for a constrained maximum is that $-g'(\chi) = (1-\omega)/\omega$. Substituting the optimal innovations into the firm's constraint and differentiating with respect to ω gives that $g'(\chi)\chi'(\omega) = \gamma'(\omega)$. So the second expression in square brackets in (A8) equals zero. On Kennedy's assumptions, then, the steady state is locally stable.

I do not know what kinds of restrictions on the innovation distribution suffice for (A7) to hold in a deterministic approximation of Duménil and Lévy's stochastic variant. Should it fail, the following argument shows that limit cycles are a possible alternative to the stable steady state. The remaining stability conditions being satisfied independently of (A7), we can take H as a bifurcation parameter of the system. If and where H passes through zero, J has a pair of purely imaginary eigenvalues whose real part is increasing in H. By Hopf's theorem there exists in this case a family of closed orbits in a neighborhood of the point of rest (Guckenheimer and Holmes (1983, pp. 151–52)). We lack the restrictions on higher-order partial derivatives that would establish that these are stable limit cycles in general by showing that they occur for H greater than zero, but simulations have found limit cycles in sample systems.

From section 6: directed technical change with local wage bargaining

This reproduces the flow just examined, but with the wage share's differential equation recast as

$$\dot{\omega} = [\psi(l) + (\mu - 1)\gamma(\omega)]\omega \tag{A9}$$

For $\mu < 1$, this change preserves the negative trace and determinant and positive pm J just found for the Jacobian of the steady state. The remaining condition for local stability becomes

$$-[n + \gamma(\omega^*)]\left[\frac{1}{1-\omega^*} + \frac{\chi'(\omega^*)}{\omega^*(1-\mu)\gamma'(\omega^*)}\right] + \chi'(\omega^*) - \gamma'(\omega^*) < 0 \tag{A10}$$

At a constrained maximum of the profit rate's time derivative expressed in (44) we have that $g'(\chi)\chi'(\omega) = \gamma'(\omega)$ and $-g'(\chi) = (1 - \omega)/(1 - \mu)\omega$, which together again ensure that the second square-bracketed term in (A10) vanishes and hence that the inequality holds. van der Ploeg (1987) finds instead that local instability is a possible regime. But he reaches this conclusion by taking the firm's first-order condition as $-g'(\chi) = (1 - \omega)/\omega$, so that the bracketed term is negative. His unstable case thus presupposes that the wage agreements implementing the passthrough are external to the individual firm or that the firm mistakenly overlooks an internal passthrough when it calculates the payoff to its prospective innovations.

REFERENCES

Acemoglu D. (2001): 'Labor- and capital-augmenting technical change', unpublished, Massachusetts Institute of Technology.

Arrow K. (1969): 'Classificatory notes on the production and transmission of technological knowledge', *American Economic Review Papers and Proceedings*, 59, pp. 29–35.

Drandrakis E., Phelps E. (1966): 'A model of induced invention, growth, and distribution', *Economic Journal*, 76, pp. 823–40.

Duménil G., Lévy D. (1995): 'A stochastic model of technical change: an application to the US economy, 1869–1989', *Metroeconomica*, 46, pp. 213–45.

Duménil G., Lévy D. (2003): 'Technology and distribution: historical trajectories *à la* Marx', *Journal of Economic Behavior and Organization*, 52, pp. 201–33.

Foley D. (2003a): 'Endogenous technical change with externalities in a classical growth model', *Journal of Economic Behavior and Organization*, 52, pp. 167–89.

Foley D. (2003b): *Unholy Trinity: Labor, Capital, and Land in the New Economy*, Routledge, London.

Funk P. (2002): 'Induced innovation revisited', *Economica*, 69, pp. 155–71.

Goodwin R. (1967): 'A growth cycle', in Feinstein C. (ed.): *Socialism, Capitalism and Economic Growth*, Cambridge University Press, Cambridge.

Guckenheimer J., Holmes P. (1983): *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, New York.

Hicks J. (1932): *The Theory of Wages*, Macmillan, London.

Julius A. (2001): 'Agent-based modeling of technical progress and capitalist economic evolution', unpublished, New School University.

Kennedy C. (1964): 'Induced bias in innovation and the theory of distribution', *Economic Journal*, 74, pp. 541–47.

Lordon F. (1995): 'Cycles et chaos dans un modèle hétérodoxe de croissance endogène', *Revue Economique*, 6, pp. 1405–25.

Lorenz H.-W. (1987): 'Strange attractors in a multisector business cycle model', *Journal of Economic Behavior and Organization*, 8, pp. 397–411.

Marglin S. (1984): *Growth, Distribution, and Prices*, Harvard University Press, Cambridge, MA.

Michl T. (1999): 'Biased technical change and the aggregate production function', *International Review of Applied Economics*, 13, pp. 193–206.

Nelson R., Winter S. (1982): *An Evolutionary Theory of Economic Change*, Harvard University Press, Cambridge, MA.

Nordhaus W. (1973): 'Some skeptical thoughts on the theory of induced technical change', Quarterly journal of economics, 87, pp. 208–19.

Okishio N. (1961): 'Technical change and the rate of profit', *Kobe University Economic Review*, 7, pp. 86–99.

Pasinetti L. (1962): 'Rate of profit and income distribution in relation to the rate of economic growth', *Review of Economic Studies*, 29, pp. 267–79.

van der Ploeg F. (1987): 'Growth cycles, induced technical change, and perpetual conflict over the distribution of income', *Journal of Macroeconomics*, 9 (1), pp. 1–12.

Samuelson P. (1965): 'A theory of induced innovation along Kennedy–Weizsäcker lines', *Review of Economics and Statistics*, 47, pp. 343–56.

Samuelson P., Modigliani F. (1966): 'The Pasinetti paradox in neoclassical and more general models', *Review of Economic Studies*, 83, pp. 269–301.

Shah A., Desai M. (1981): 'Growth cycles with induced technical change', *Economic Journal*, 91, pp. 1006–10.

Skillman G. (1997): 'Technical change and the equilibrium profit rate in a market with sequential bargaining', *Metroeconomica*, 48, pp. 238–61.

Skott P. (1981): 'Technological advance with depletion of innovation possibilities: a comment and some extensions', *Economic Journal*, 91, pp. 977–87.

Steedman I. (1973): 'Some long-run equilibrium tax theory', *Public Finance*, 28 (1), pp. 43–51.

Thompson F. (1995): 'Technical change, accumulation, and the profit rate', *Review of Radical Political Economics*, 27, pp. 97–126.

von Weizsäcker C. C. (1966): 'Tentative notes on a two-sector model with induced technical progress', *Review of Economic Studies*, 95, pp. 245–52.

2125 Holland Avenue
Apartment 6H
Bronx
NY 10462
USA
E-mail: julia056@newschool.edu